4. Unsinkable Disk

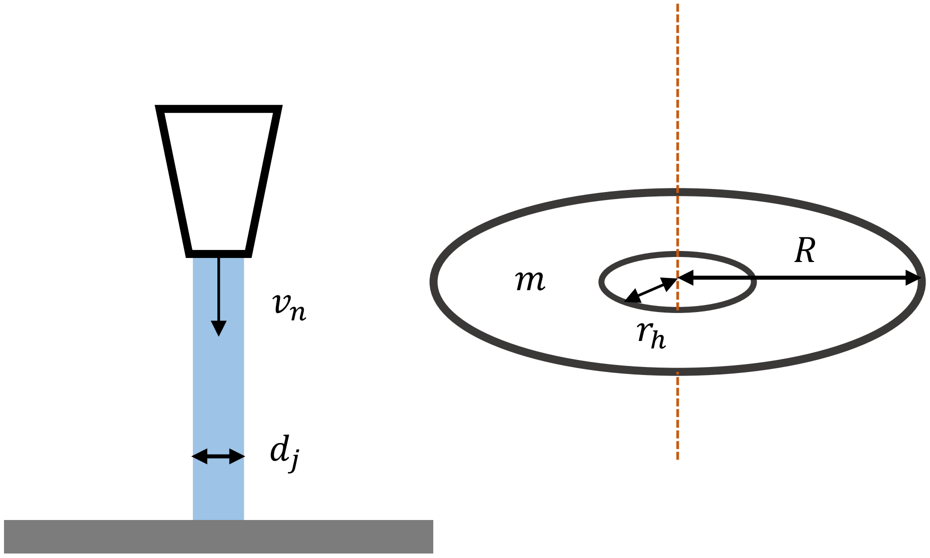
"A metal disk with a hole at its centre sinks in a container filled with water. When a vertical water jet hits the centre of the disc, it may float on the water surface. Explain this phenomenon and investigate the relevant parameters."

Problem Statement

*"A metal disk with a **hole** at its centre **sinks** in a container filled with **water**. When a **vertical water jet** hits the **centre of the disc**, it may **float** on the water surface. **Explain** this phenomenon and investigate the **relevant parameters**."*

Parameters:

1. Jet speed
2. Jet diameter
3. Hole radius
4. Disk radius
5. Disk mass



Overview

Phenomenon Cases

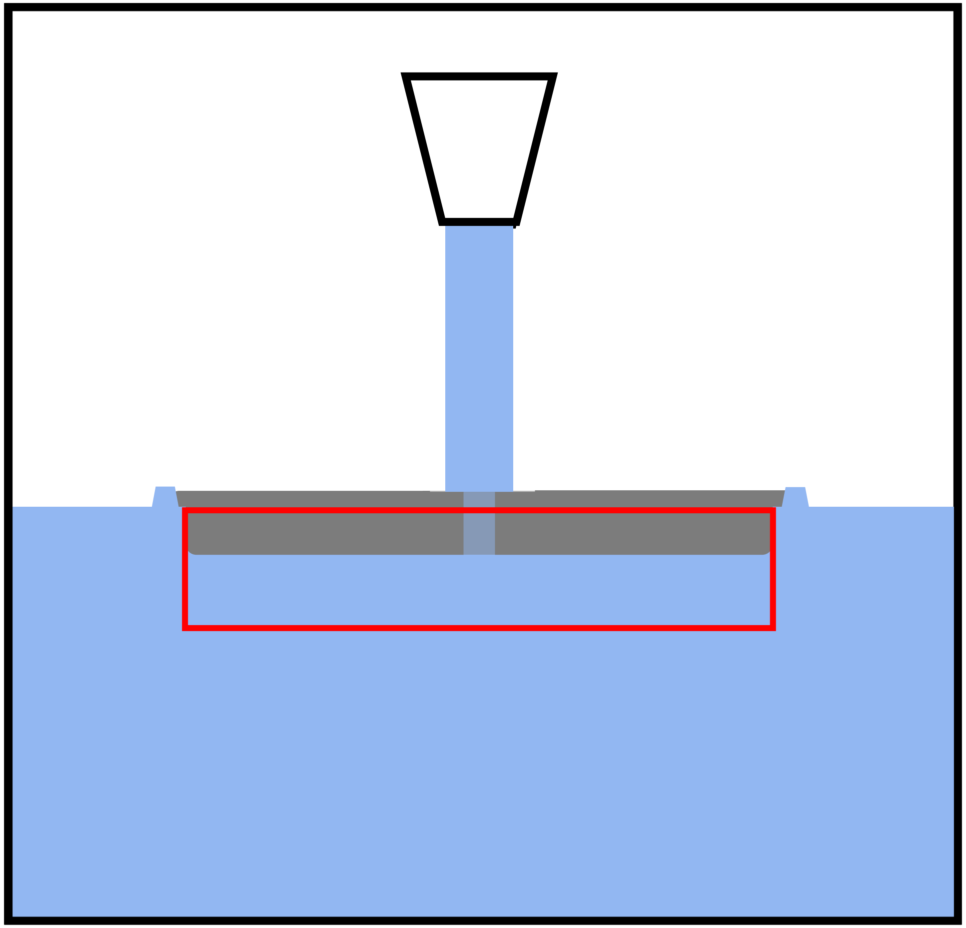
The Unsinkable Disk has two distinct cases

Case 1:

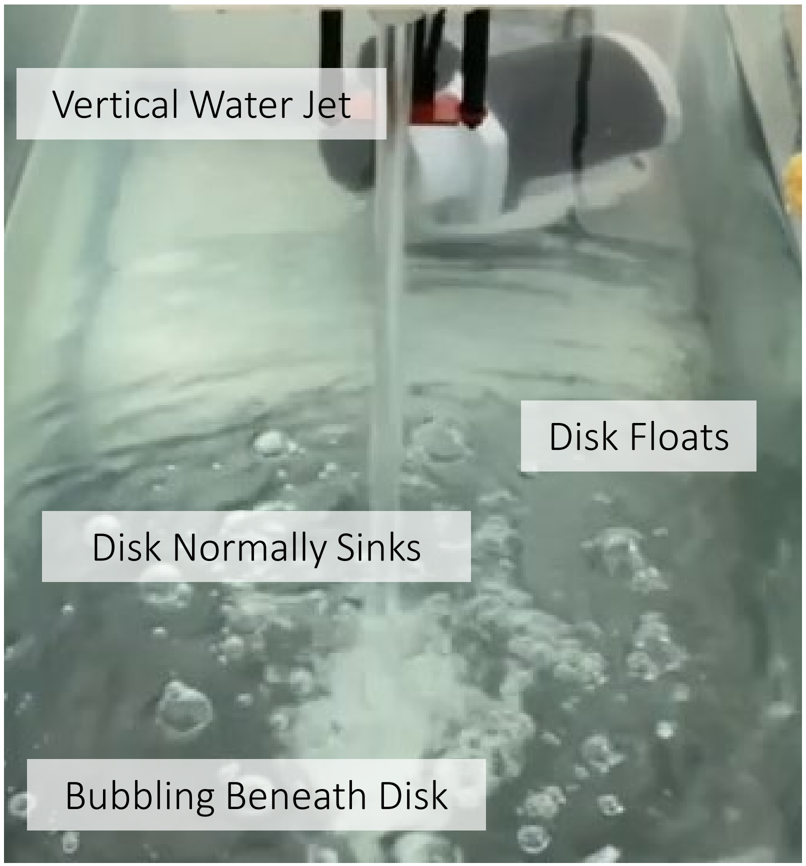
$Hole\ Radius > Jet\ Radius$

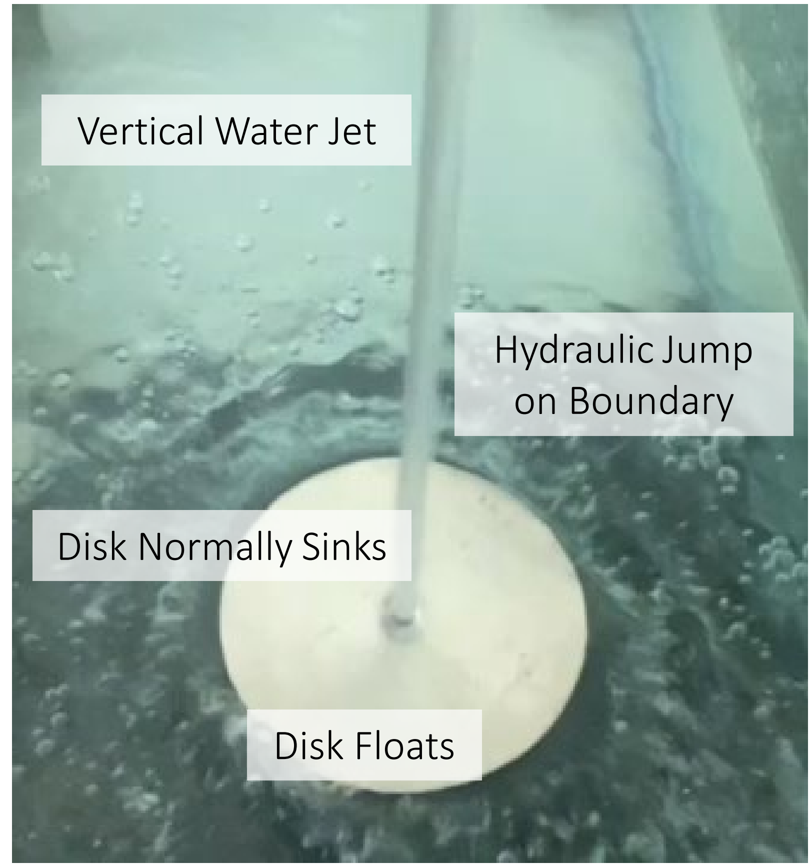
Case 2:

$Hole\ Radius < Jet\ Radius$



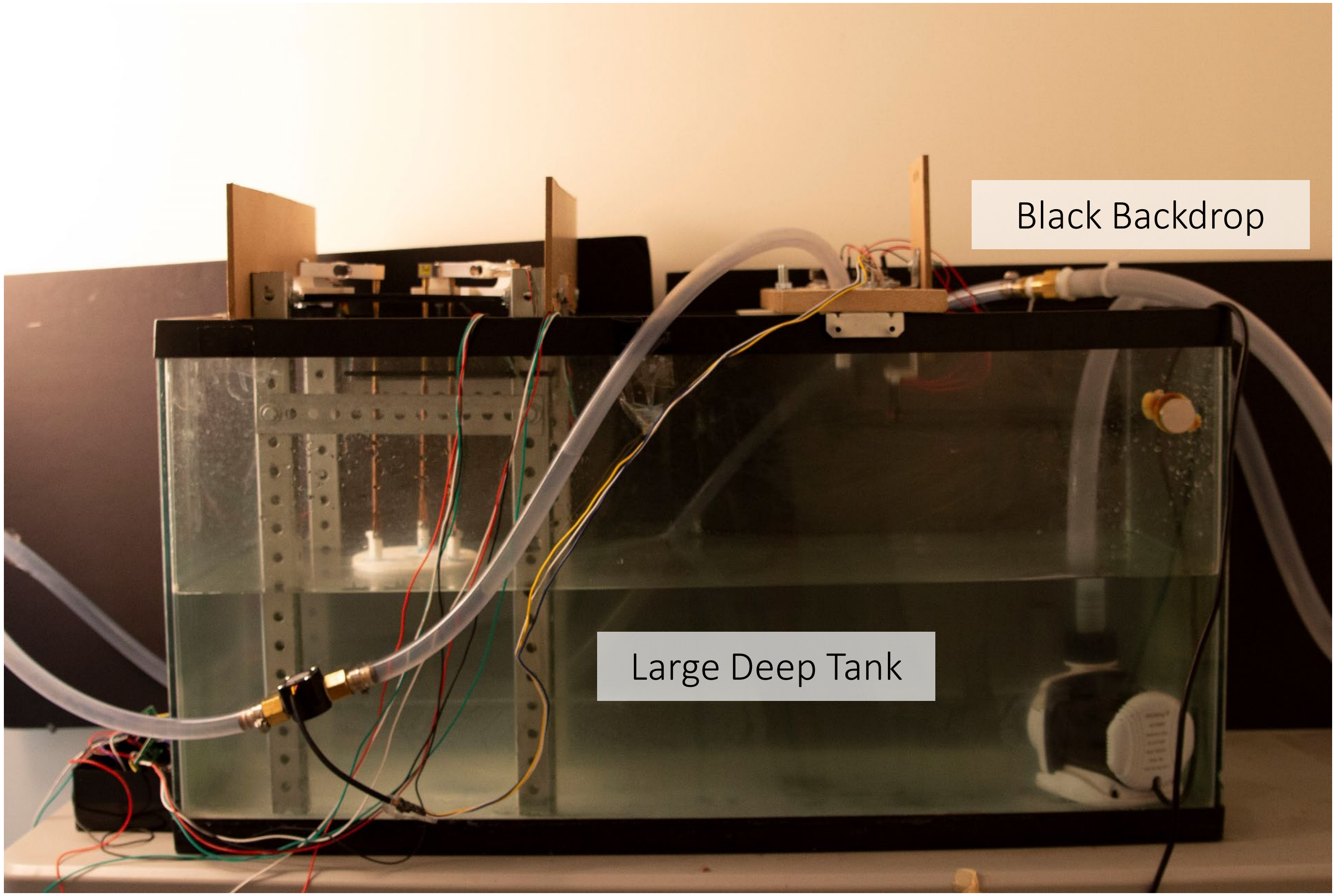
Phenomenon


Vertical Water Jet
Disk Floats
Disk Normally Sinks
Bubbling Beneath Disk

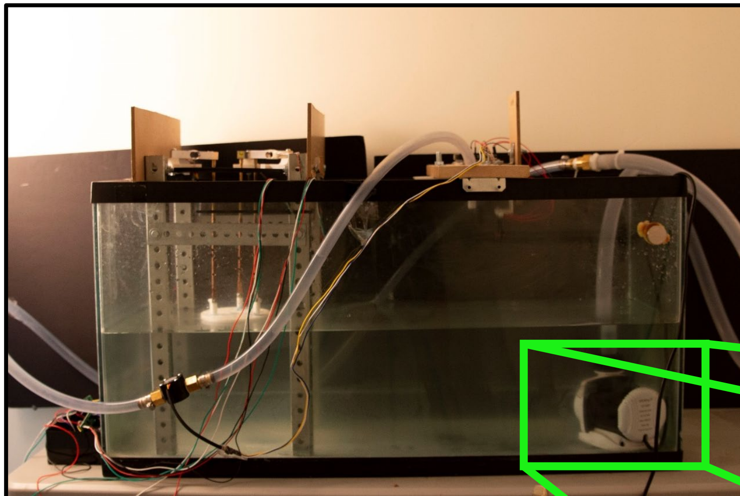

Vertical Water Jet
Hydraulic Jump on Boundary
Disk Normally Sinks
Disk Floats

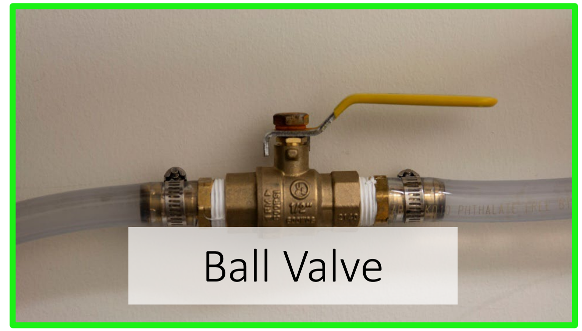
Experimental Setup

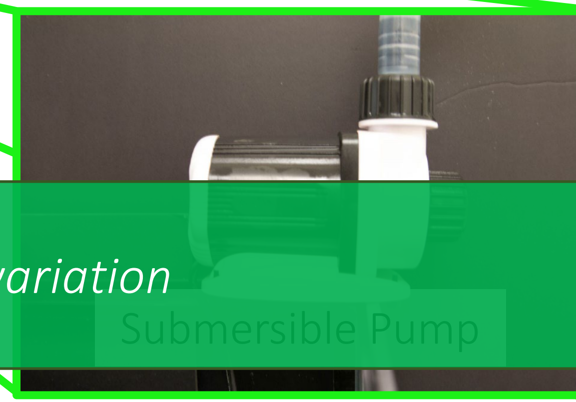
Experimental Setup



Pump







Specifications:

DC Aquarium Pump

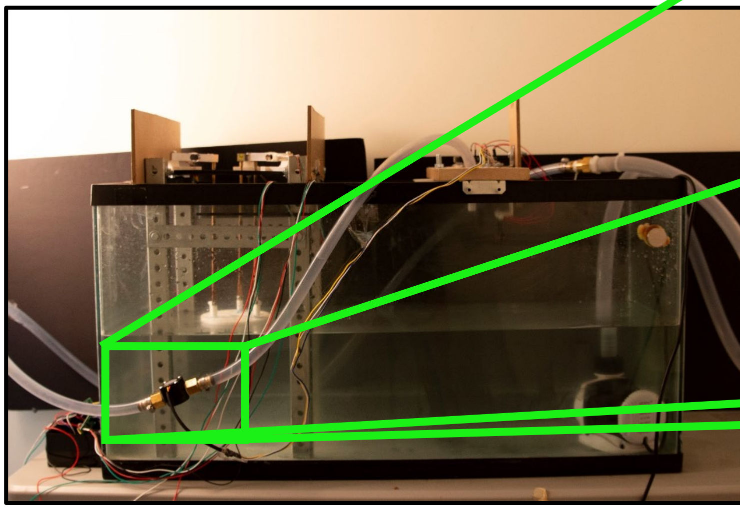
20 Adjustable Flow Rates With Remote

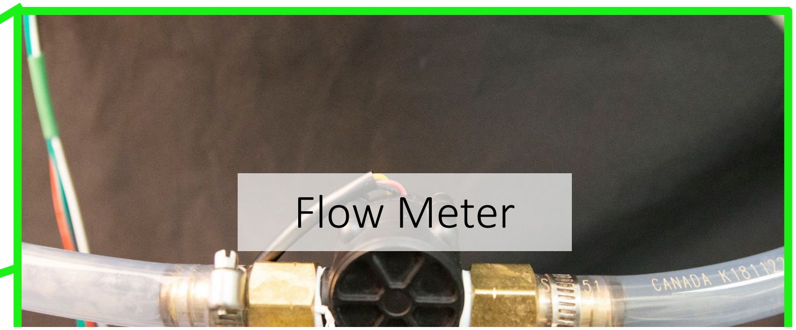
< 80mL/s with Ball Valve

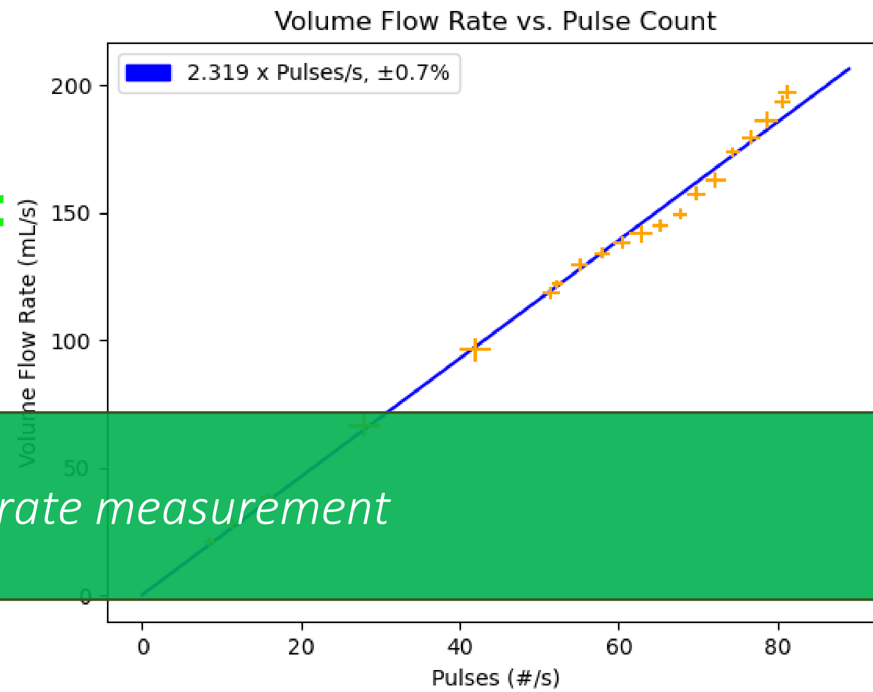
Up to 3260 mL/s

✓ *Controlled flow rate variation*

Flow Meter







Specifications:

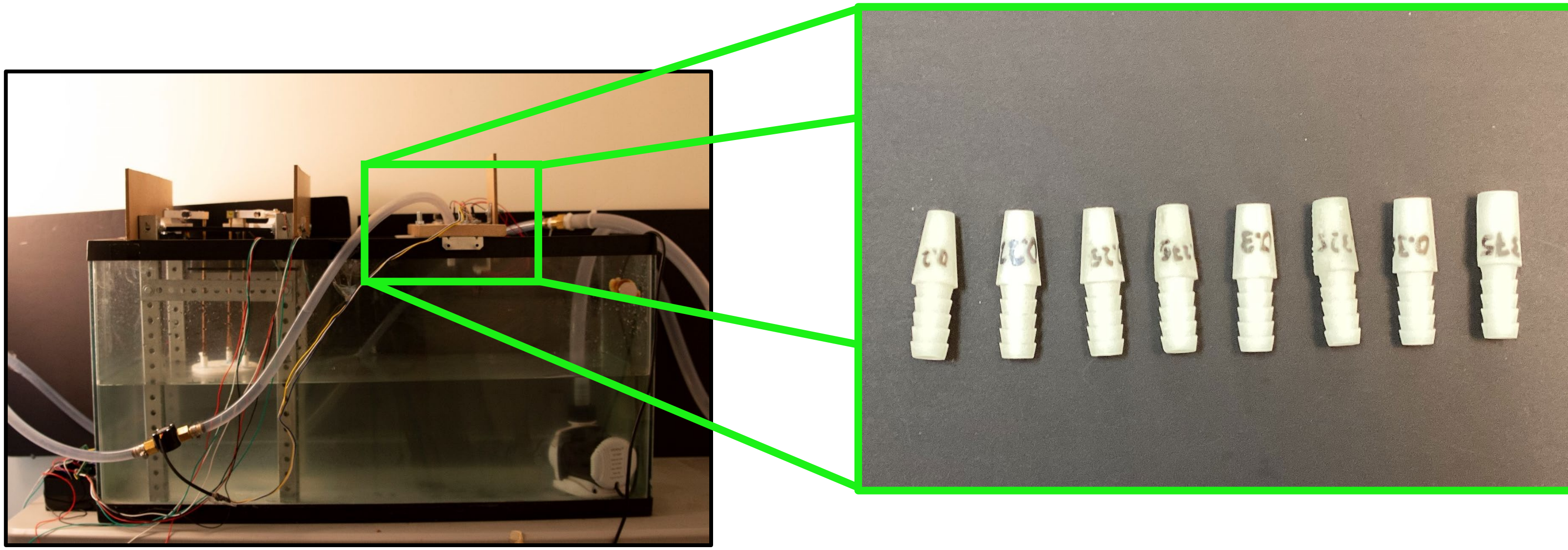
DIGITEN Water Flow Sensor

17mL/s – 500mL/s

Accurate flow rate measurement

Pulse Counter with Arduino

Nozzles



Specifications:

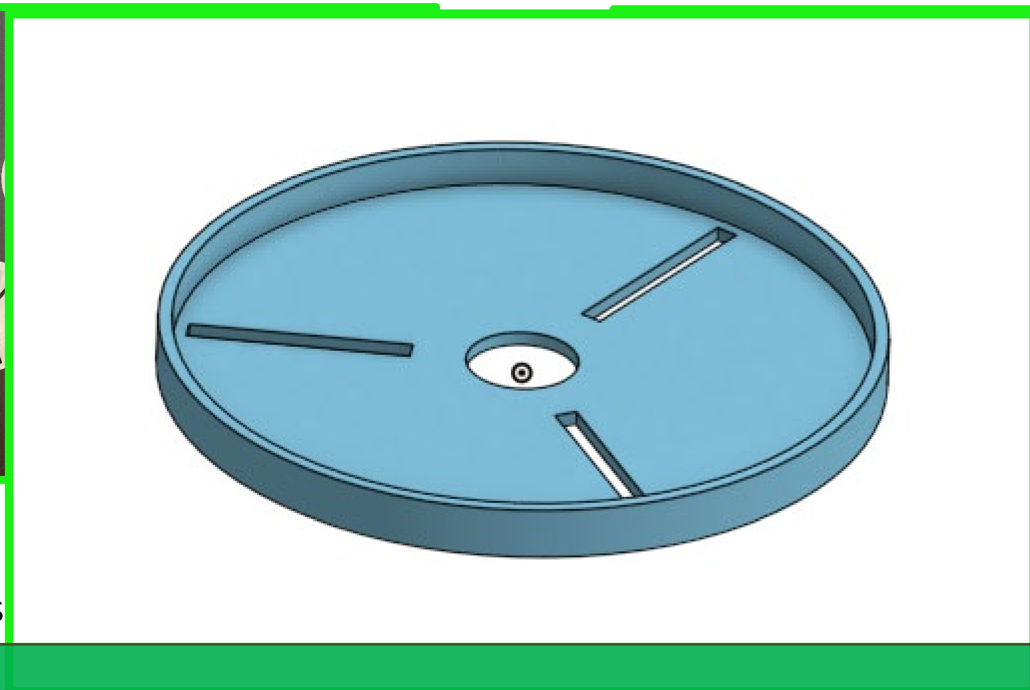
7 Different nozzle diameters

SLA 3d Printer - Resin

Controlled jet radius change

0.375in. – 0.200in. diameter

Disks



Specifications

7 different ho

6 different disk diameters 8.00cm – 16.00cm

Added rim beneath disk to add d vary mass

FDM 3d Printer – PLA

Clay to vary disk mass

Clay to close slits

Controlled change of mass, radius and hole radius

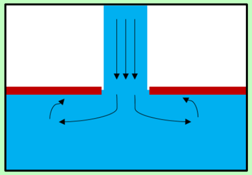
Theoretical Model

Theoretical Model

A

Flow Dynamics (hole>jet)

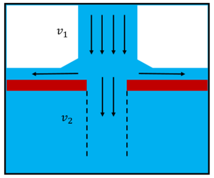
Force Analysis, Jet Effects, Empirical Model



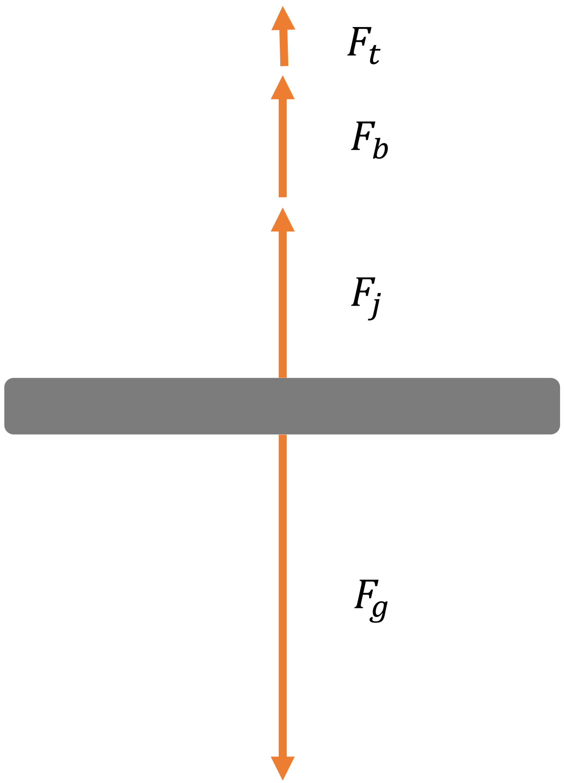
B

Hydraulic Jump (jet>hole)

Impinging Force, Archimedes' Principle, Empirical Model



Free Body Diagram



$F_g = Force\ of\ gravity$

$F_j = Force\ of\ jet$

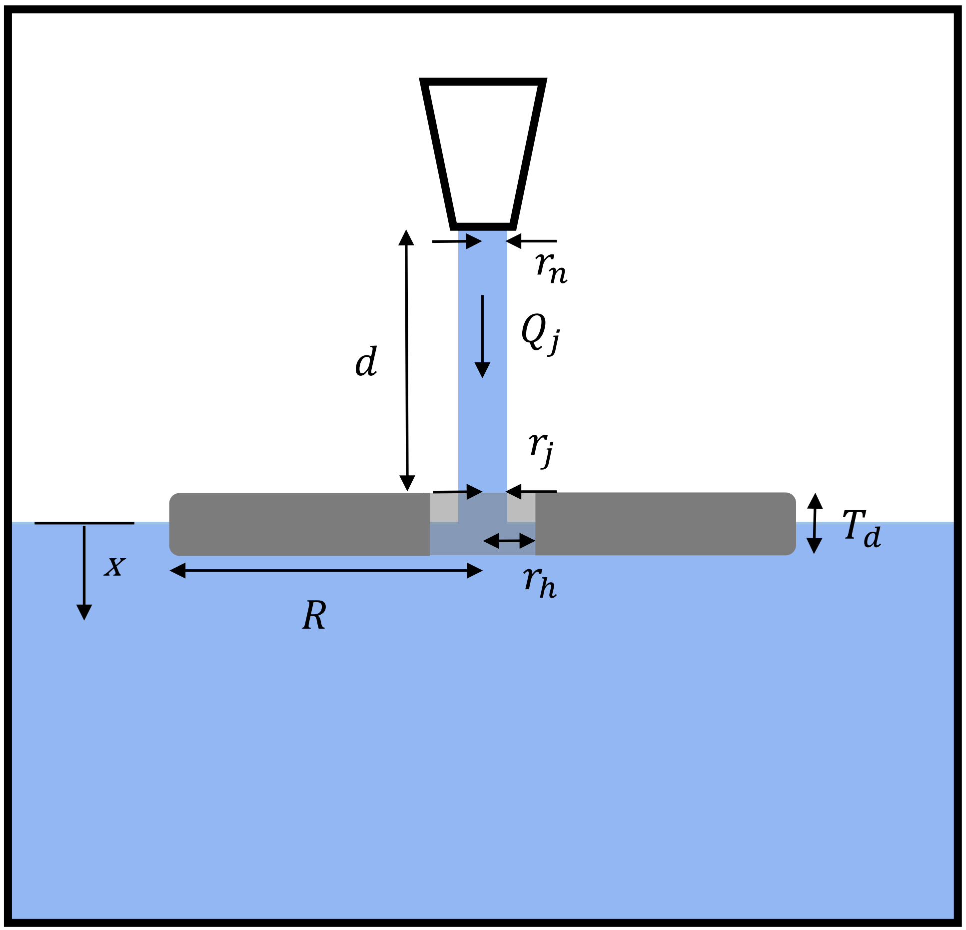
$F_b = Buoyant\ force$

$F_t = Force\ of\ surface\ tension$

Equilibrium necessary to float:

$$F_g - F_b - F_t - F_j = 0$$

Geometry



Terms:

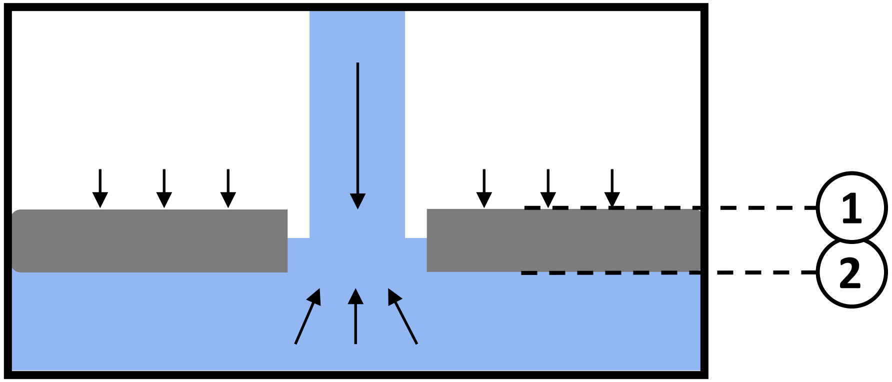
r_n = nozzle radius
r_j = jet radius
r_h = hole radius
R = disk radius
T_d = disk thickness
d = nozzle height
Q_j = volume flow rate

Assumptions

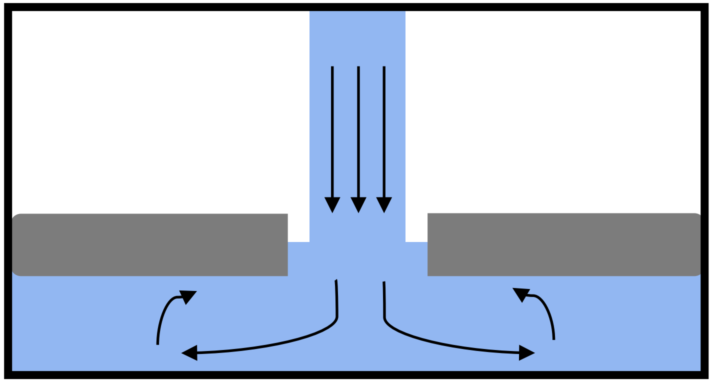
- ! Jet does not collide with edges of disk
- ✓ Jet remains vertical, centered, and at constant velocity
- ✓ Tank is large enough to neglect wave effects
- ✓ Water is incompressible

Jet Forces

Decompose jet forces into two components



Opposing flow through hole



Additional vortices and gas

Through Bernoulli's equation:

$$\boxed{P_{\text{atm}}} + \rho g T_d = \boxed{P_{\text{atm}}} + \boxed{\rho g x} + \frac{1}{2}\rho v^2$$

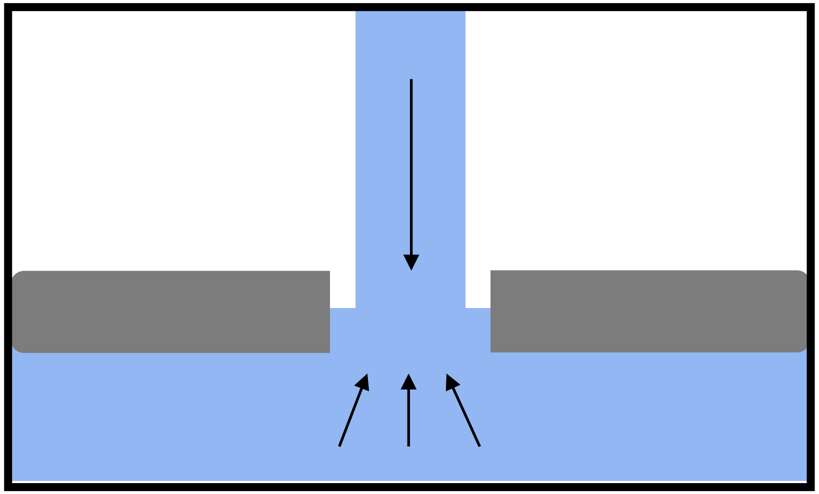
Through continuity:

$$\pi R^2 \dot{x} = \pi r^2 v$$

$$\dot{x} = \sqrt{2g(T_d - x)}\left(\frac{r}{R}\right)^2$$

Potential Flow

For disks that barely dense enough for sinking, jet flow only opposes potential flow



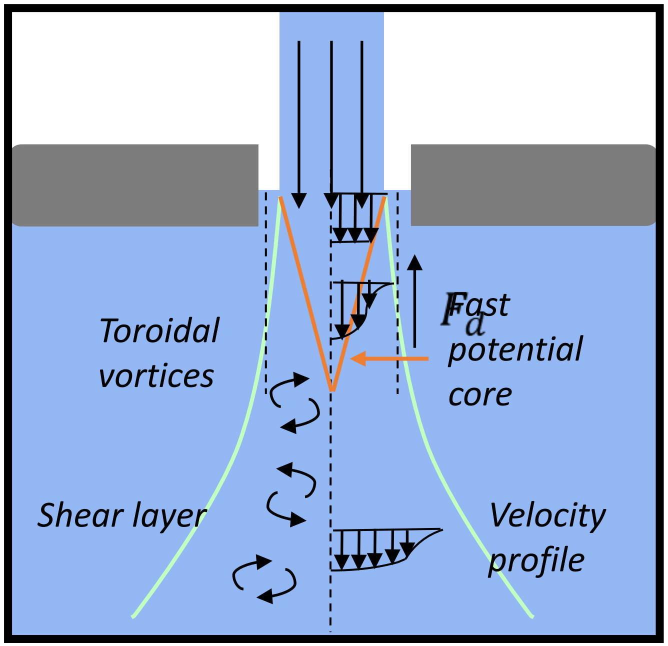
$$v = \dot{x} = \sqrt{2g(T_d - x)}\left(\frac{r}{R}\right)^2$$

Calculating minimum flow rate needed from v:

$$Q = \pi r^2 \sqrt{2g(T_d - x)}\left(\frac{r}{R}\right)^2$$

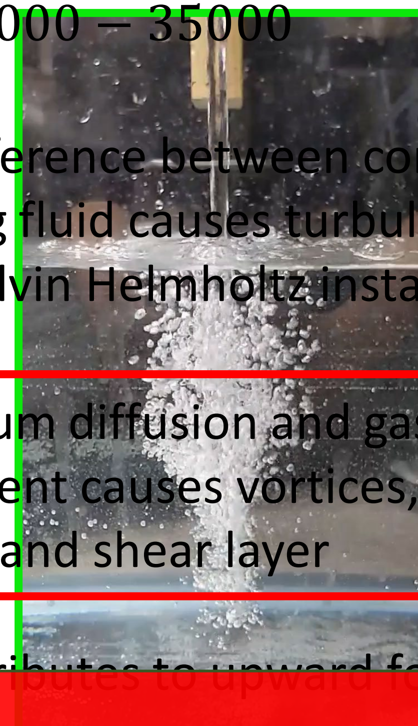
! *Additional jet forces must be present for heavier disks*

Vortices and Turbulence



After impact, jet continues to flow while slowed by drag force reaching terminal depth

We calculate:
$Re: 25000 - 35000$



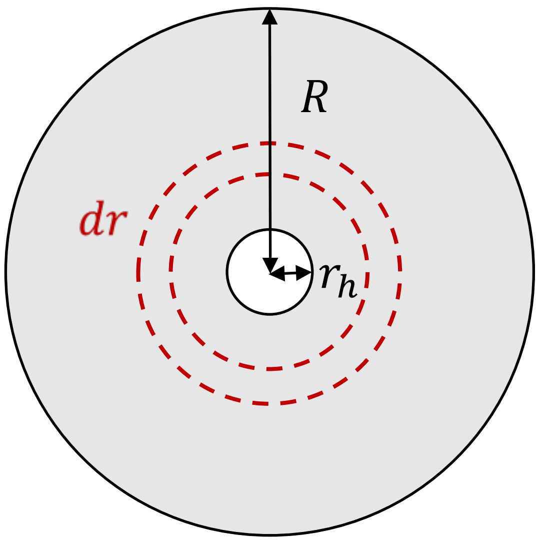
Velocity difference between core and surrounding fluid causes turbulence and vortices (Kelvin Helmholtz instability)

Momentum diffusion and gas entrainment causes vortices, bubbling and shear layer

Contributes to upward force

Complexity of combined jet forces requires empirical modelling

Empirical Force Field



Consider upwards forces applied to disk by jet as a non-uniform pressure field over area

$$\boldsymbol{P}(\boldsymbol{R}, \boldsymbol{v_j}, \boldsymbol{r_j})$$

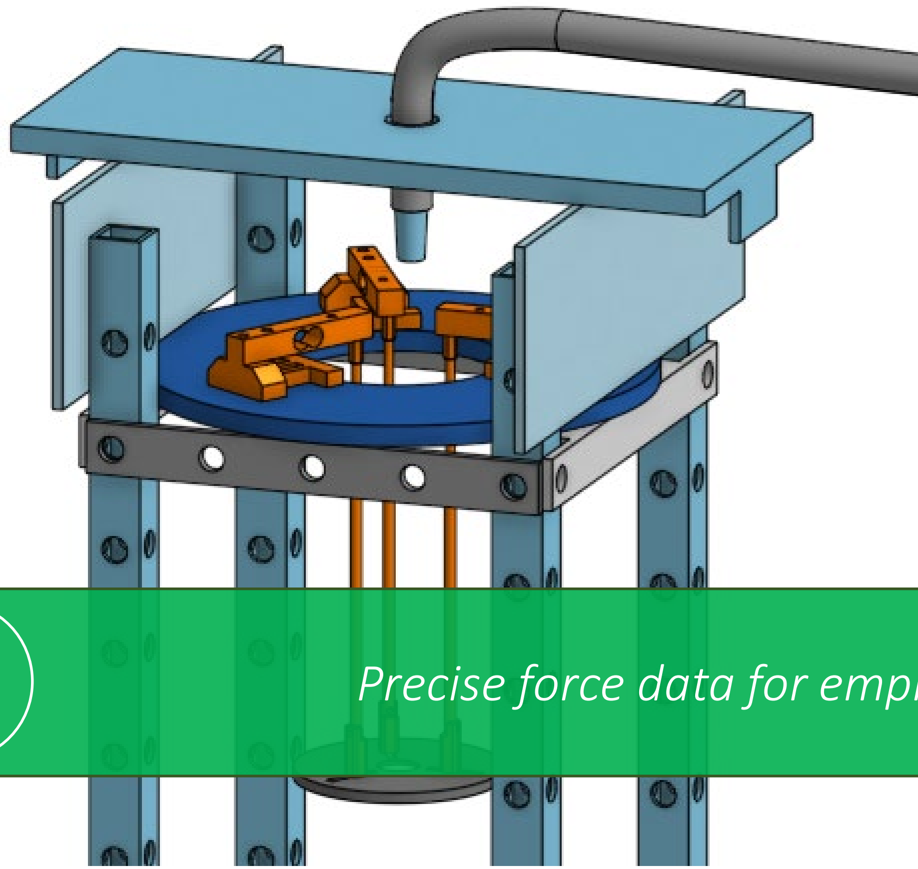
$$F_j = 2\pi \int_{r_h}^{R} P\left(R, v_j, r_j\right) r\,dr$$

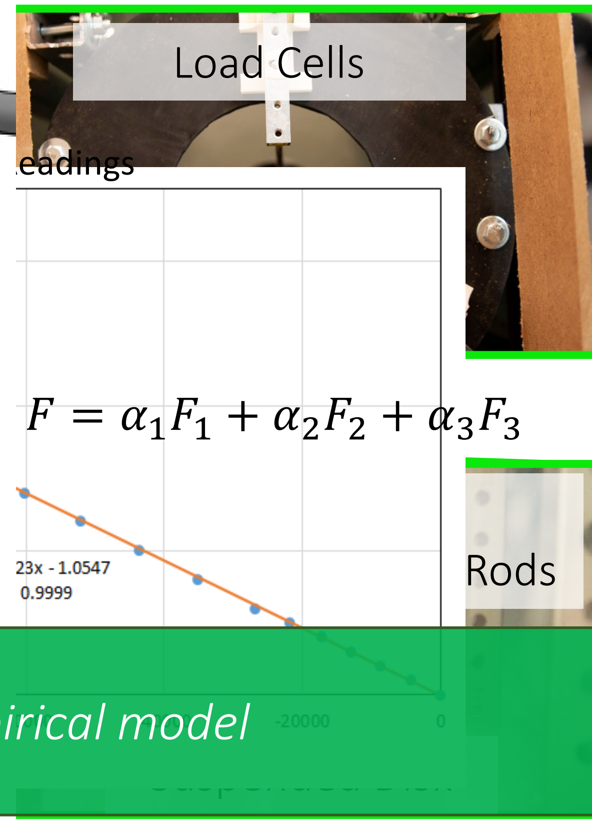
Q_j

Area integral yields overall force acting on disk

Experimentally isolate each parameter to visualize pressure field

Force Measurement System

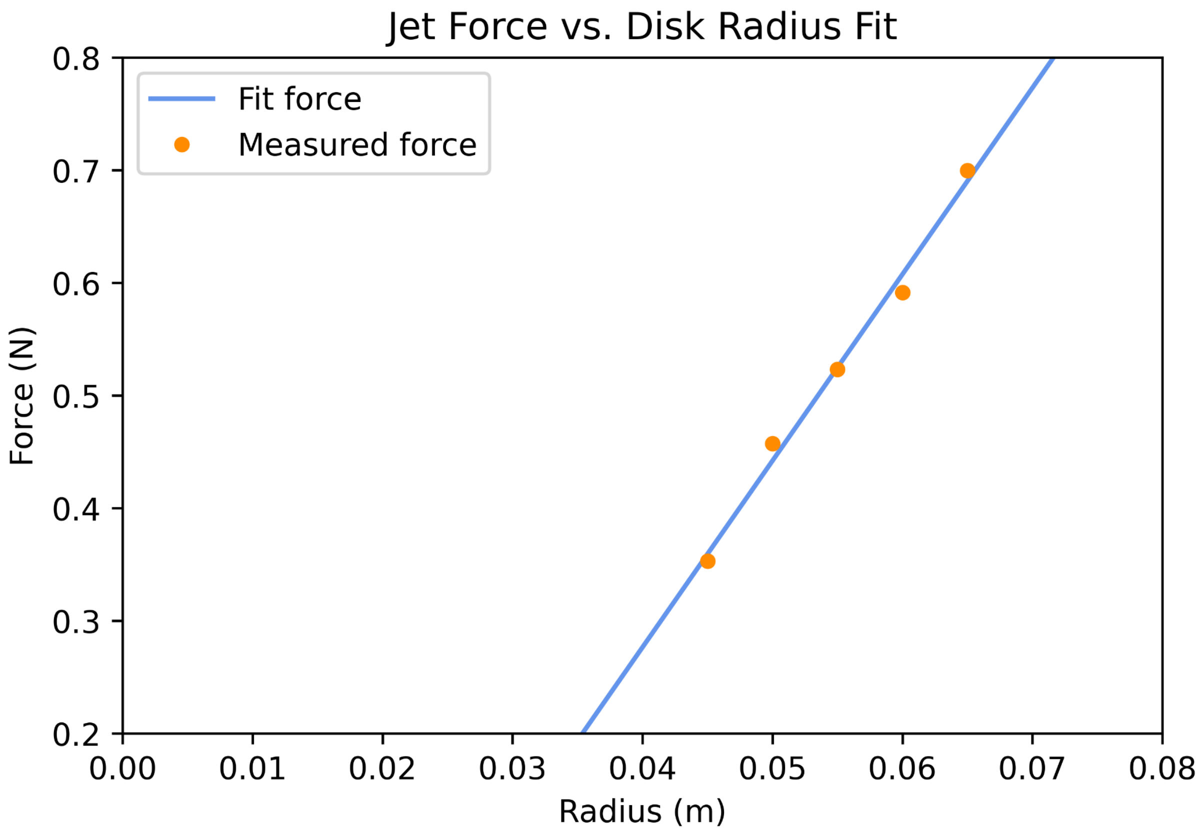


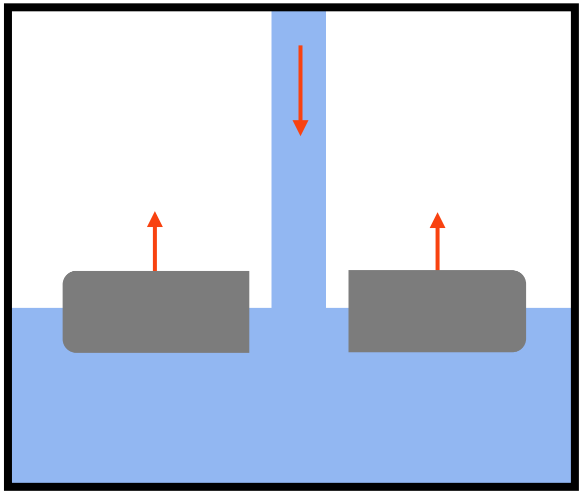


$$F = \alpha_1 F_1 + \alpha_2 F_2 + \alpha_3 F_3$$

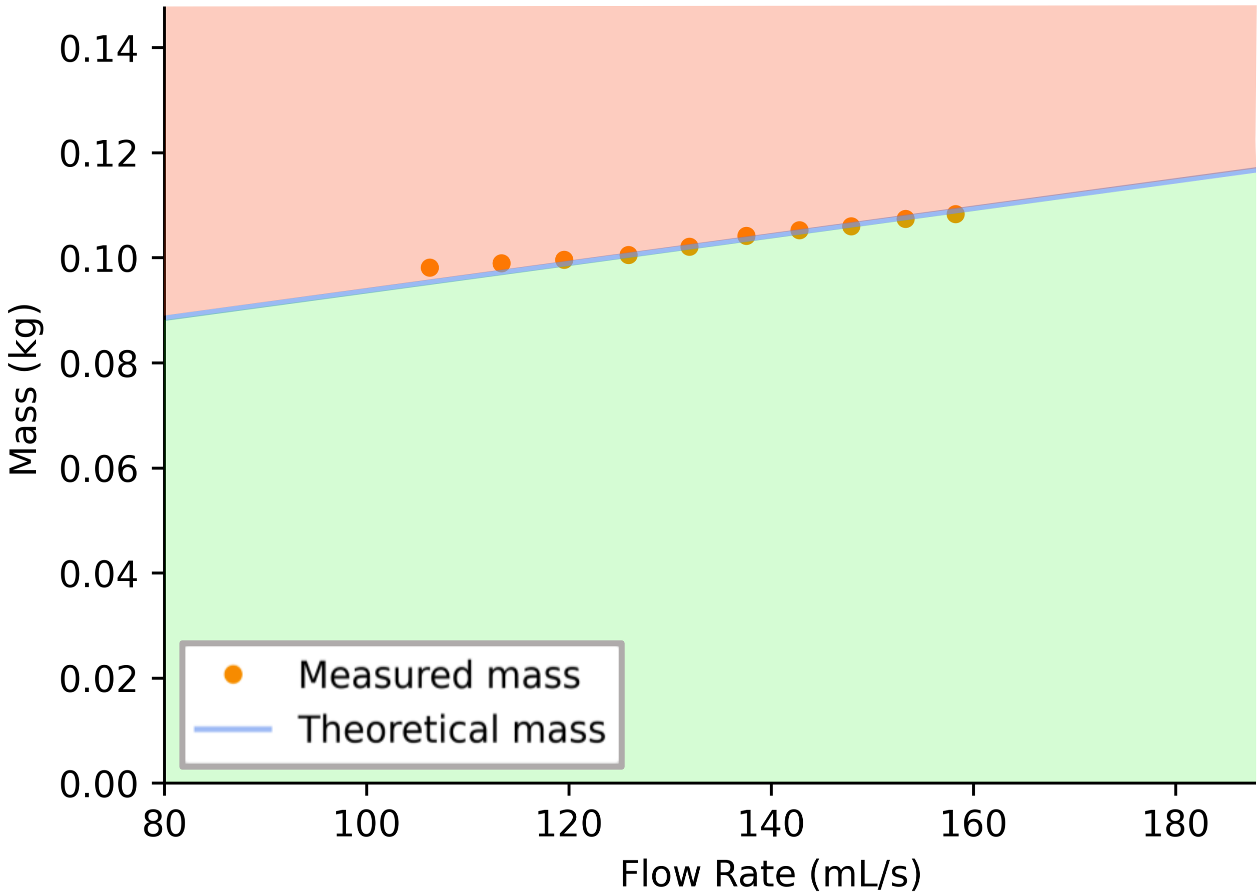
Precise force data for empirical model

Empirical Force Fit



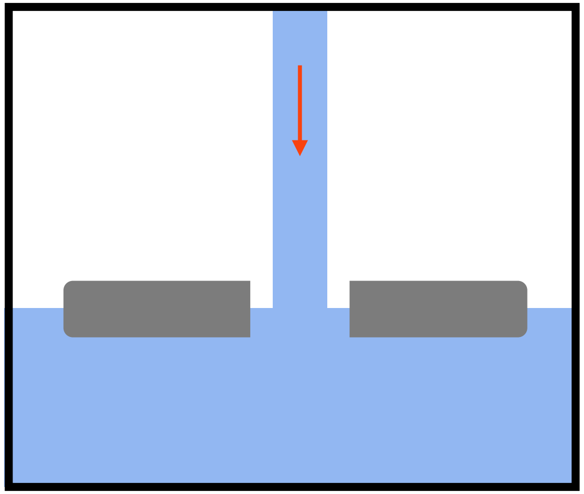


Experimental Verification



Equilibrium Equation

$$F_g - F_b - F_t - F_j = 0$$

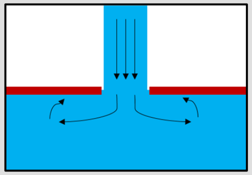


Theoretical Model

A

Flow Dynamics

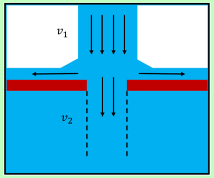
Force Analysis, Jet Effects, Empirical Model



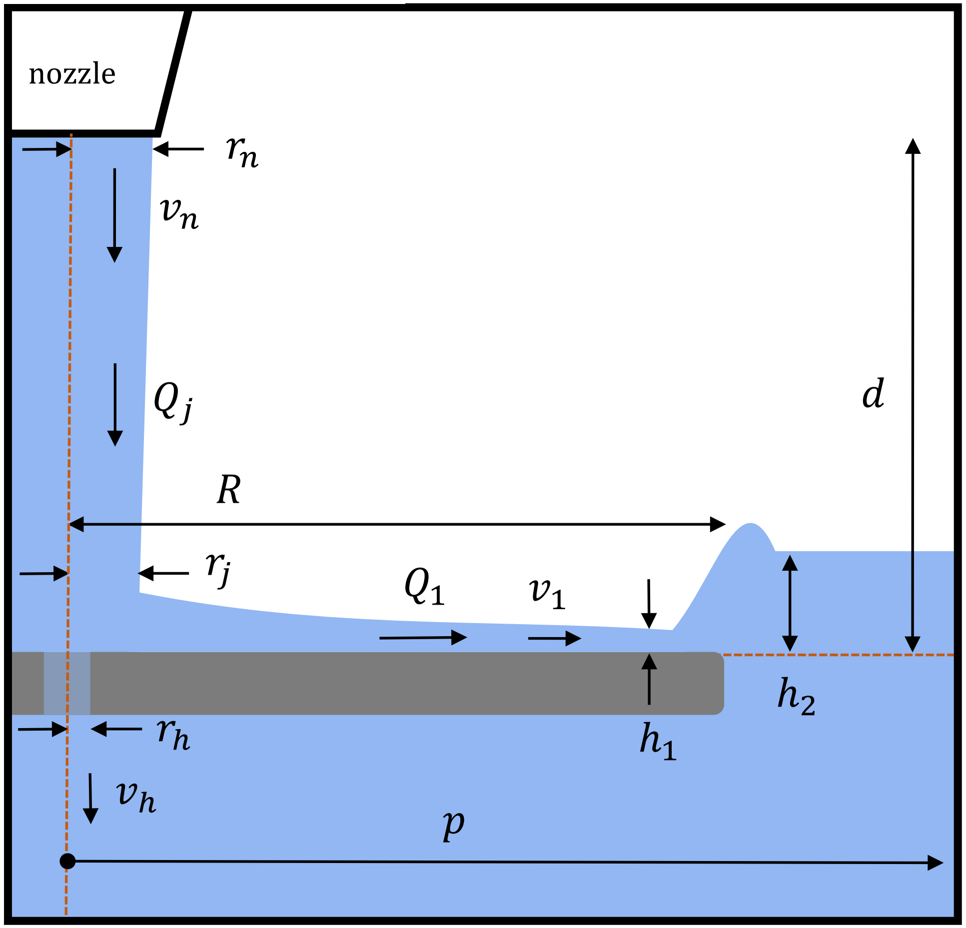
B

Hydraulic Jump

Impinging Force, Archimedes' Principle, Empirical Model



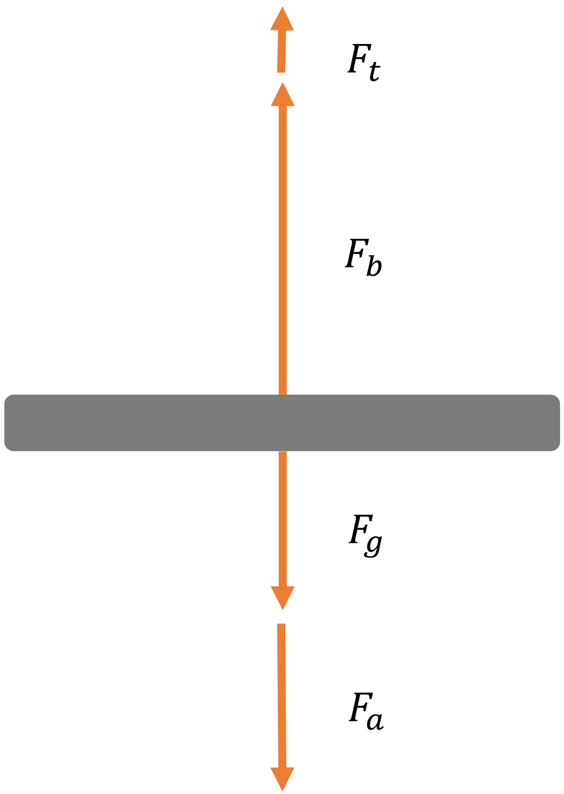
Geometry



Terms:

Q = *volume flow rate*
$\dot{m}$ = *mass flow rate*
r = *radius*
v = *fluid velocity*
h = *fluid height*
d = *nozzle height*
p = *radial coordinate*
m = *mass*

Free Body Diagram



$F_g = Force\ of\ gravity$

$F_a = Impinging\ force\ of\ jet$

$F_b = Buoyant\ force$

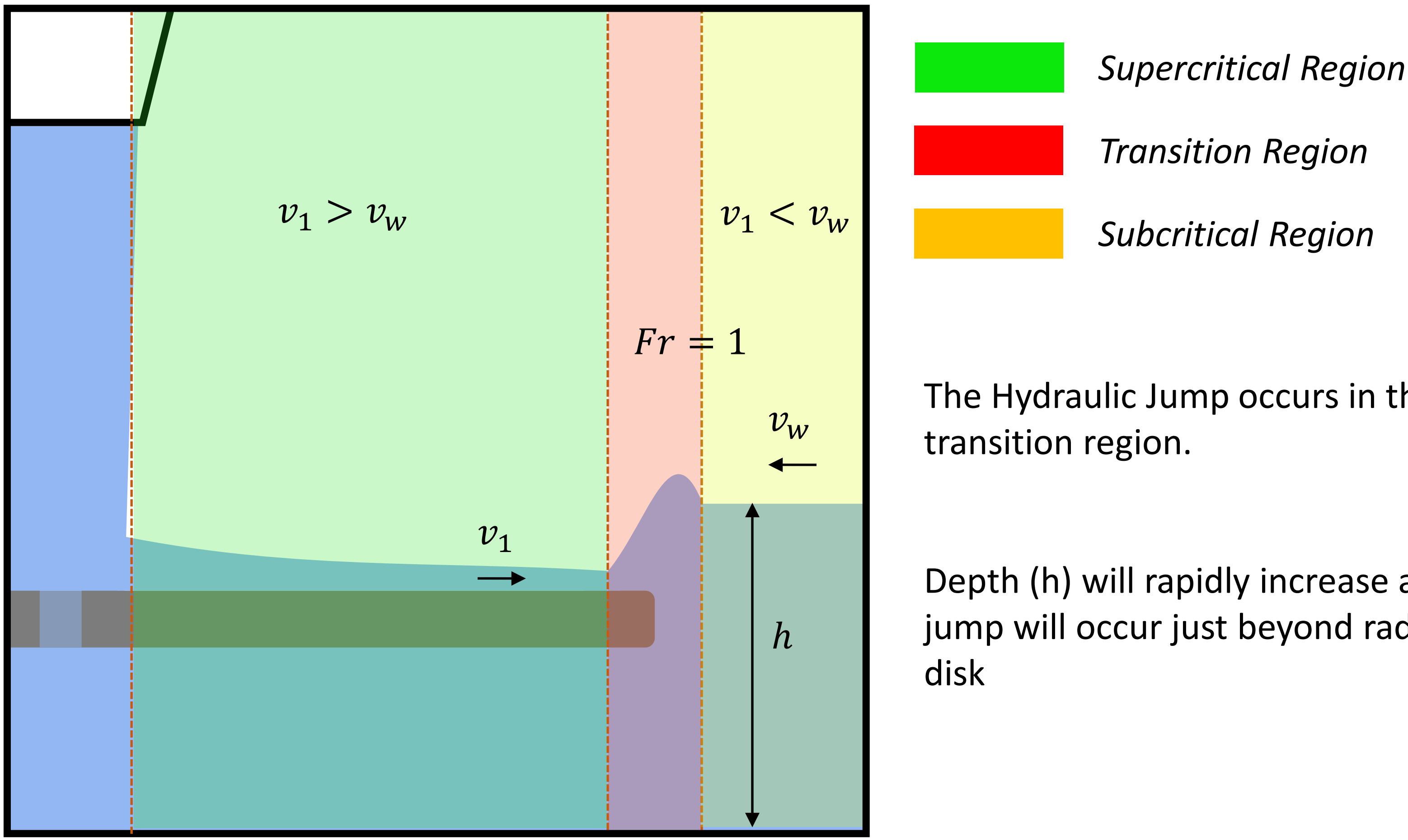
$F_t = Force\ of\ surface\ tension$

Equilibrium necessary to float:

$$F_g + F_a - F_b - F_t = 0$$

Hydraulic Jump

$$v_w = \sqrt{gh}$$



The Hydraulic Jump occurs in the transition region.

Depth (h) will rapidly increase and the jump will occur just beyond radius of disk

Additional Force

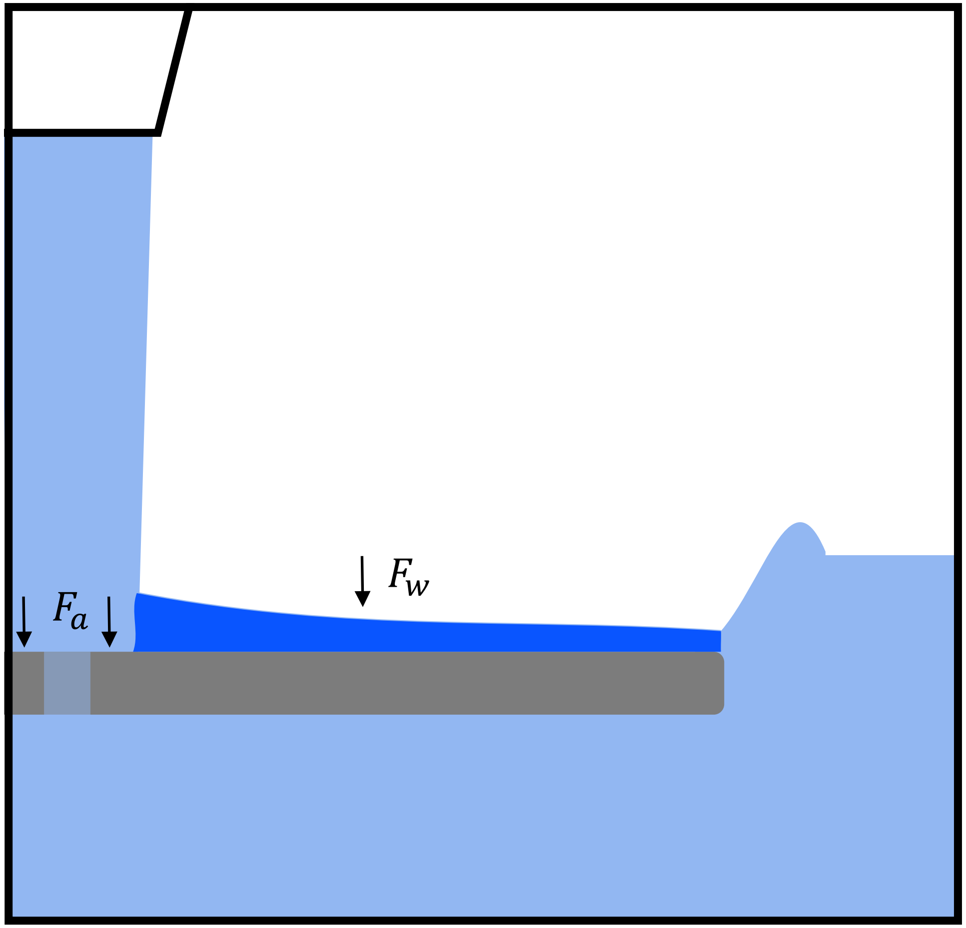
Terms:
$F_a = impinging\ force$
$F_w = force\ of\ gravity\ of\ water$

New force equilibrium equation:

$$2\pi R\gamma\cos(\theta) + \rho g V = mg + F_w + F_a$$

Surface Tension; Buoyant Force; Weight; Water Weight; Impinging Force

Solve For F_w, F_a, and V



Narrowing Jet

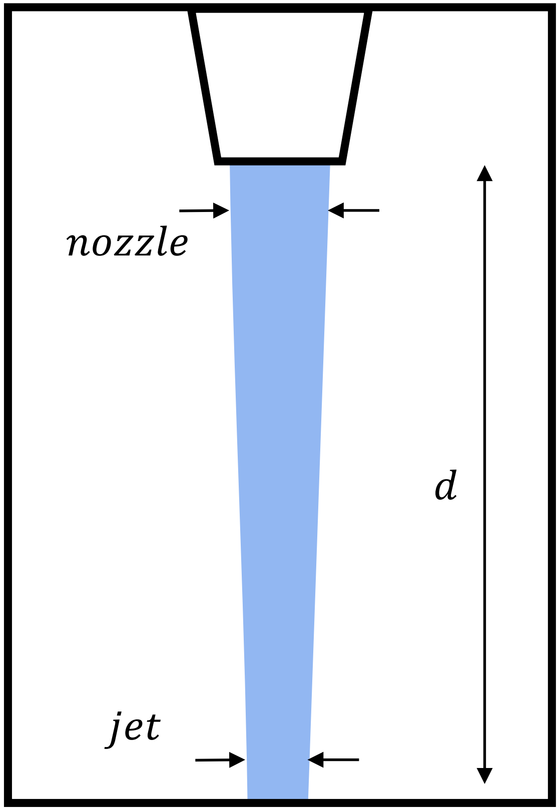
The dimensions of the jet will change after accelerating for distance d

Use Bernoulli's Principle to find velocity:

$$v_j = \sqrt{v_n^2 + 2gd}$$

Use Continuity to find radius:

$$r_j = \sqrt{\frac{Q_j}{v_j\,\pi}}$$



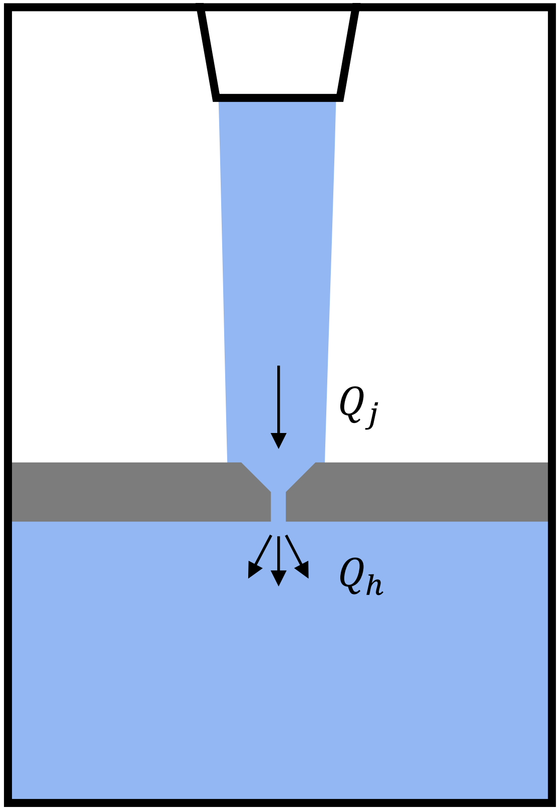
Hole Flow Rate

In order for the system to be continuous, the flow rate escaping through the hole must be found.

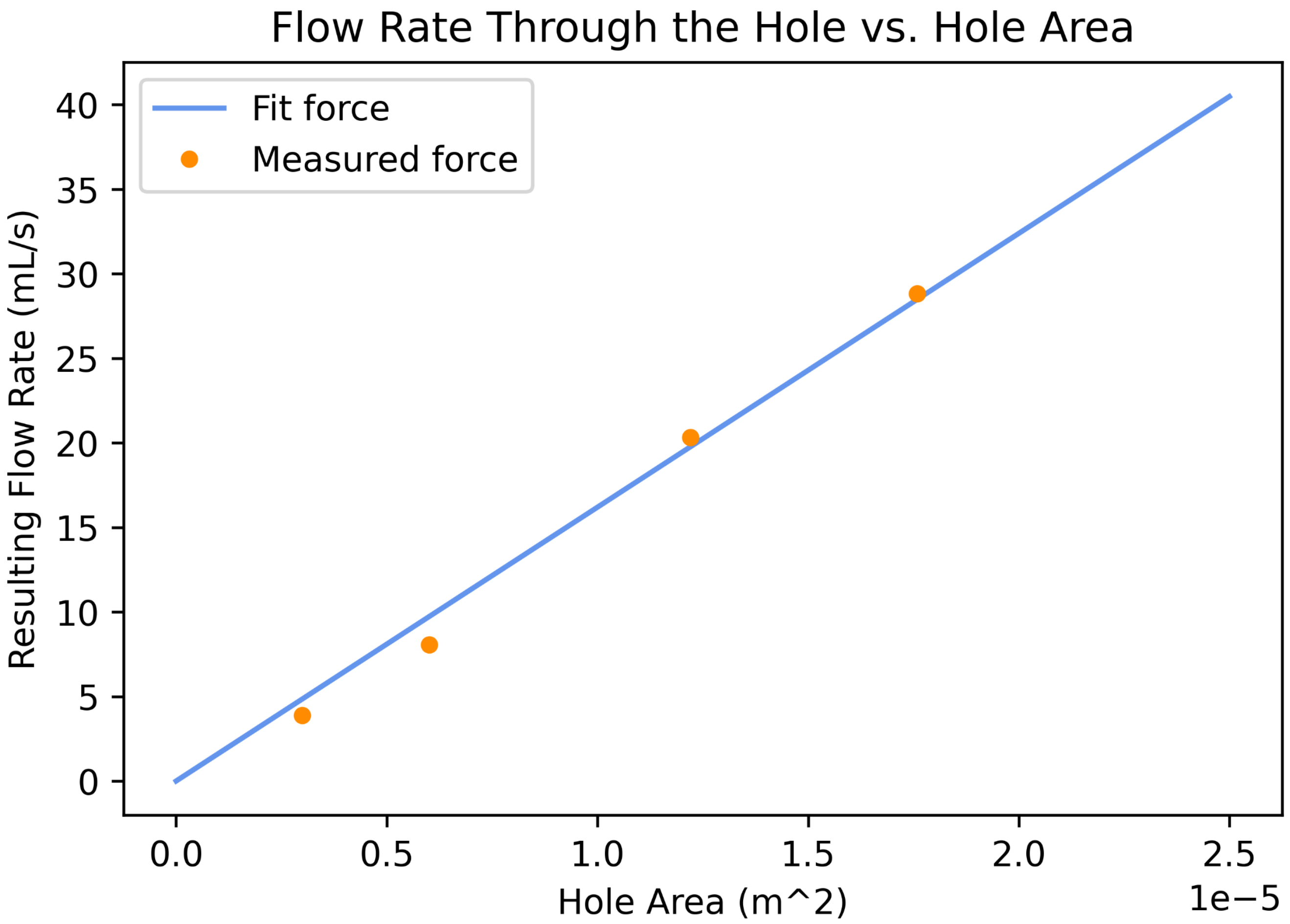
Due to the indent in the disk for stability, this is very hard to model.

We will rely on an empirical fit

$$Q_h(Q_j, \frac{r_h}{r_j})$$



Hole Flow Rate



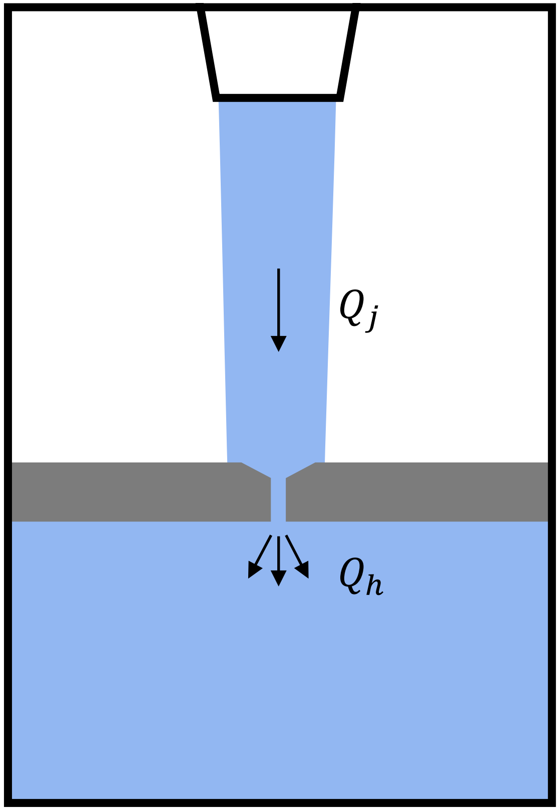
Impinging Force

To find the force of the jet hitting the plate, we can solve for linear momentum. Using the previously fitted mass flow rate through the hole

$$F_a = \dot{m}_j v_j - \dot{m}_h v_h$$

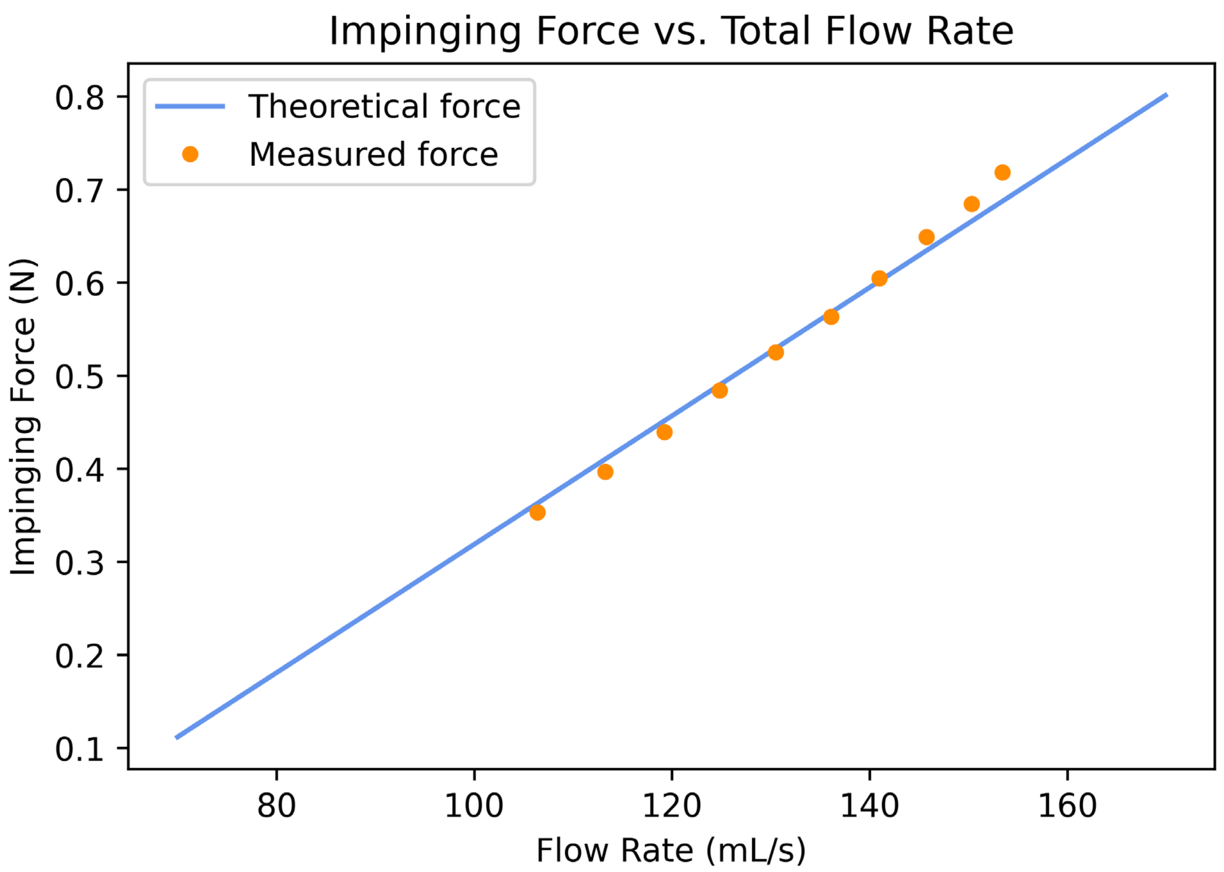
$$F_a = v_j^2 \rho A_j - v_h^2 \rho A_h$$

$$F_a = \rho(Q_j v_j - Q_h v_h)$$



Impinging Force Experimental

Using Force Measurement System



Water Weight

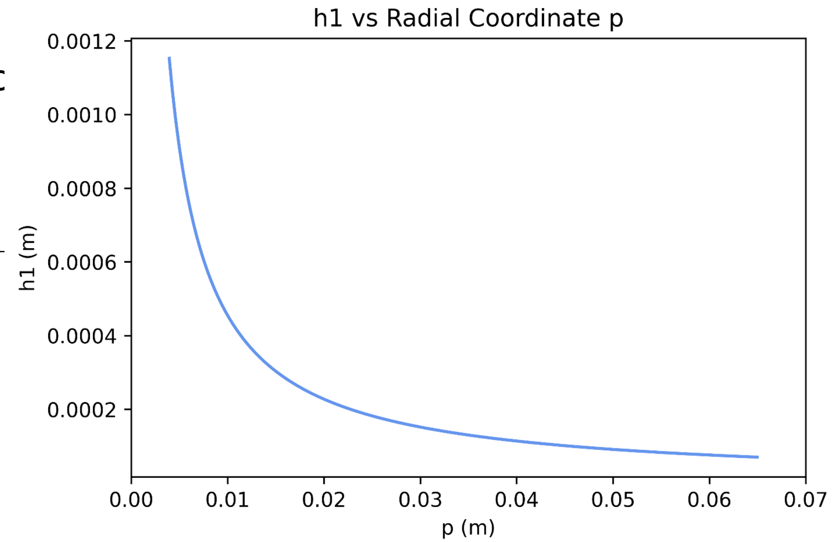
Similar to the Jet Force Field, we can find h_1 as a func
area integrals to find water volume.

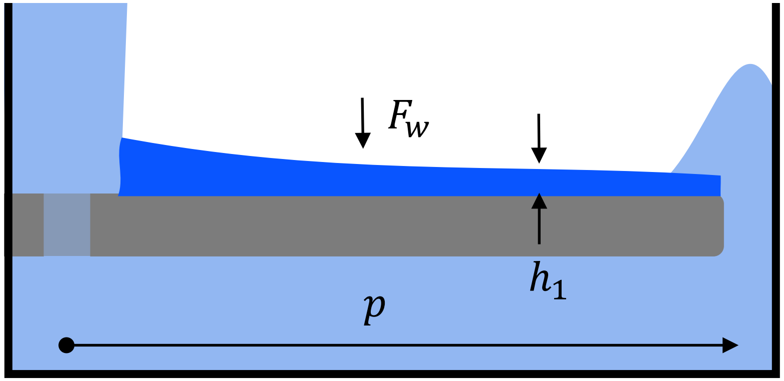
Assuming the Hydraulic Jump Radius is larger than the disk radius, we can derive an equation for height from energy conservation and continuity:

$$h_1 = \frac{(Q_j - Q_h)^{\frac{3}{2}}}{p\ 2\pi\sqrt{Q_j(v_n^2 + 2gd) - Q_h v_h^2}}$$

$$V = 2\pi\int_{r_h}^{R} h_1(p)dp = \frac{(Q_j - Q_h)^{\frac{3}{2}}}{\sqrt{Q_j(v_n^2 + 2gd) - Q_h v_h^2}}(R - r_h)$$

$$F_w = \rho g V$$

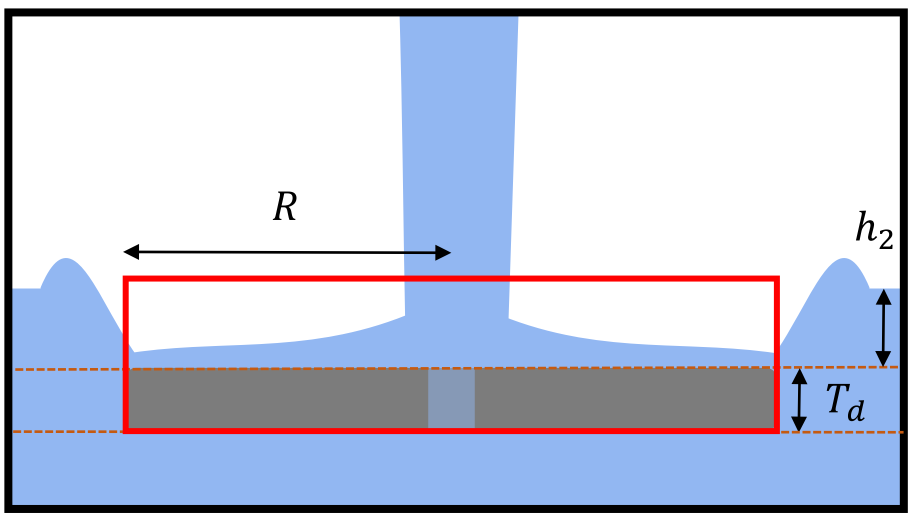


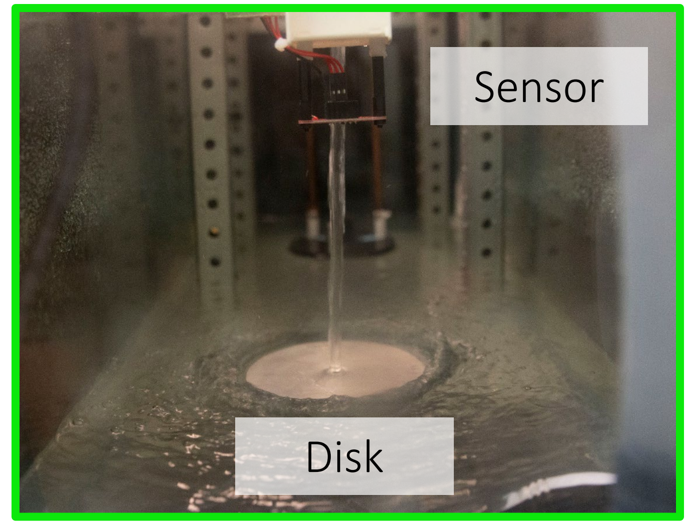


Buoyant Force - Range Sensor

Buoyant force ∝ the depth of the disk below the water surface (h_2).

$$F_b = \rho g V = \rho g(\pi R^2 - \pi r_h^2)(h_2 + T_d)$$





Specifications:

VL6180 Time-of-Flight Range Sensor

0.000m – 0.250m ± 0.0005m

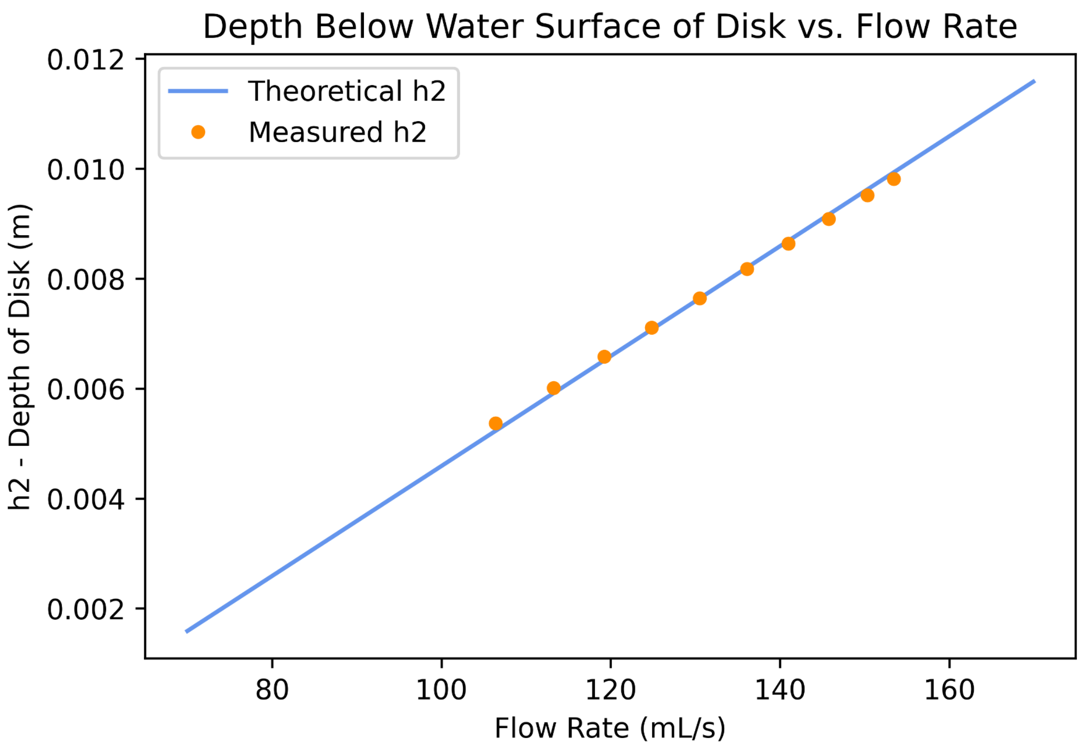
To measure change in height of the disk

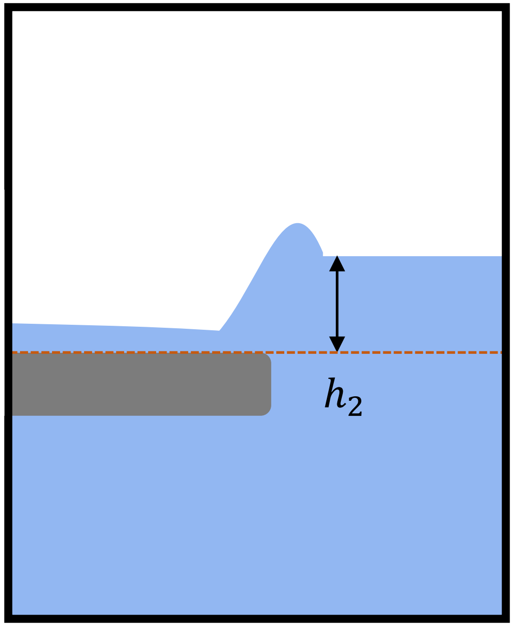
Height Solution

Applying the equilibrium equation, we predict a value h_2 and compare to the Range Sensor Data

Equilibrium Equation

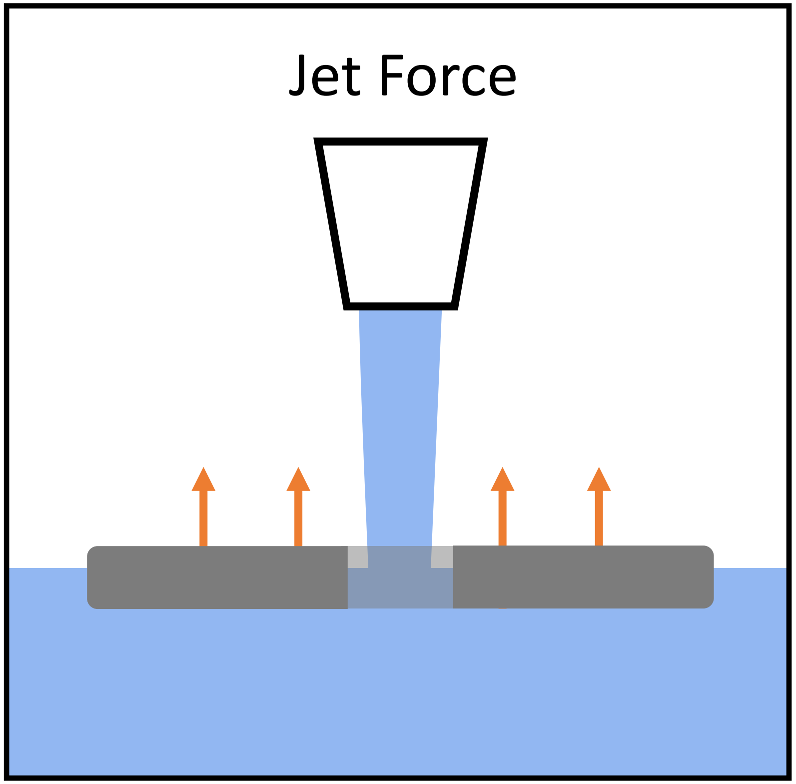
$$2\pi R\gamma \cos(\theta) + \rho g V = mg + F_w + F_a$$

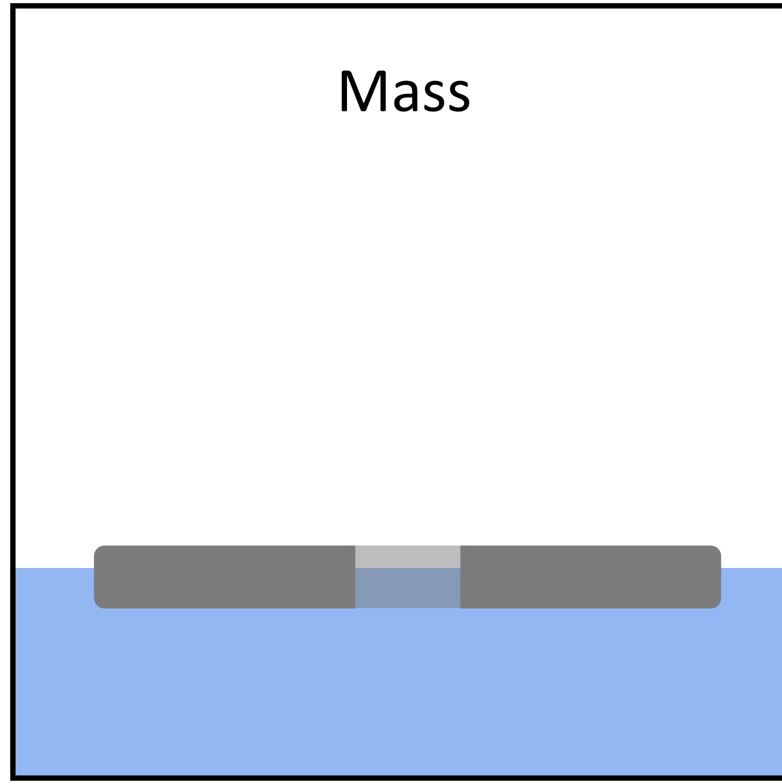




Key Parameters

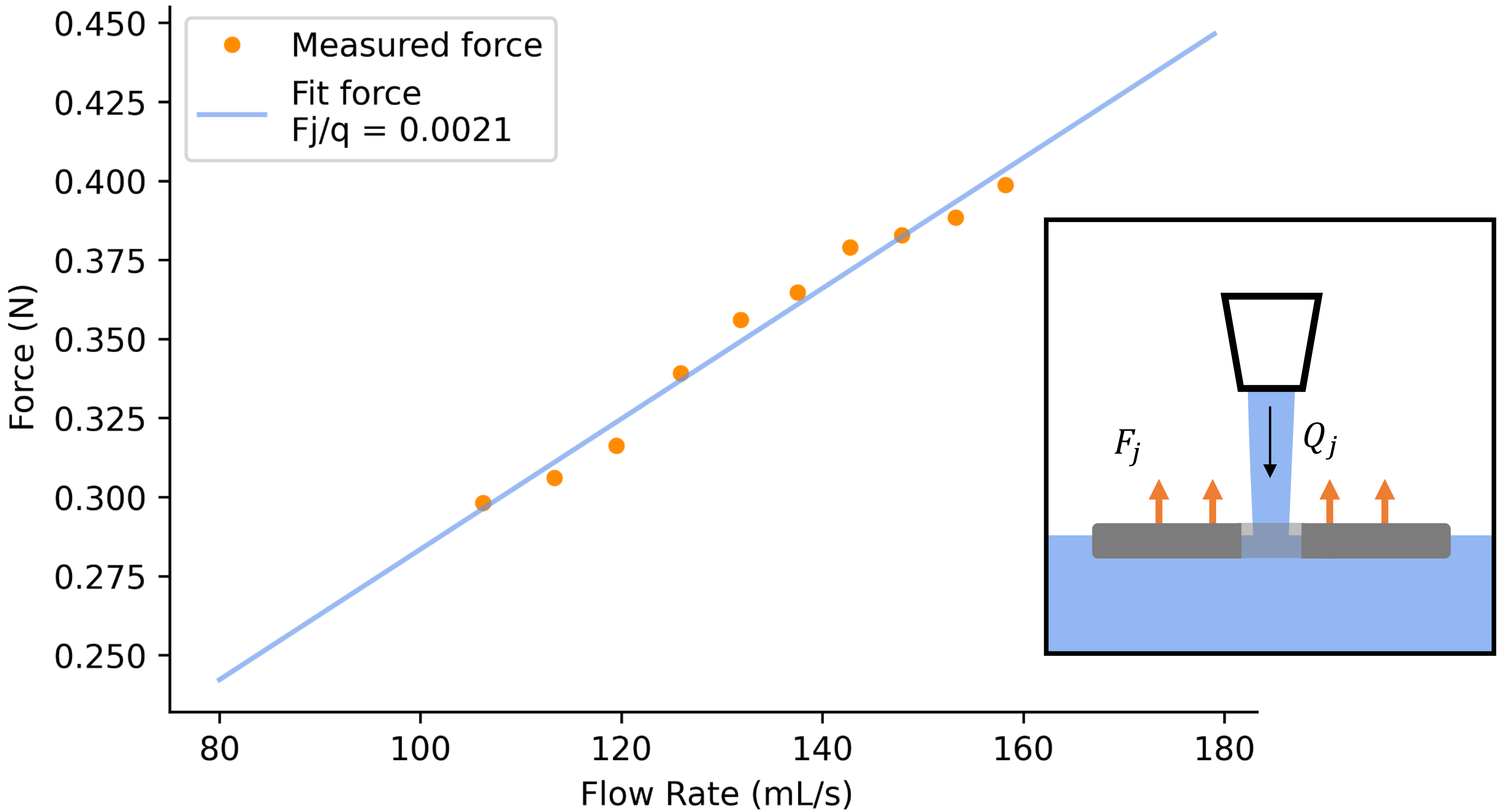
Key Parameters – Flow Dynamics



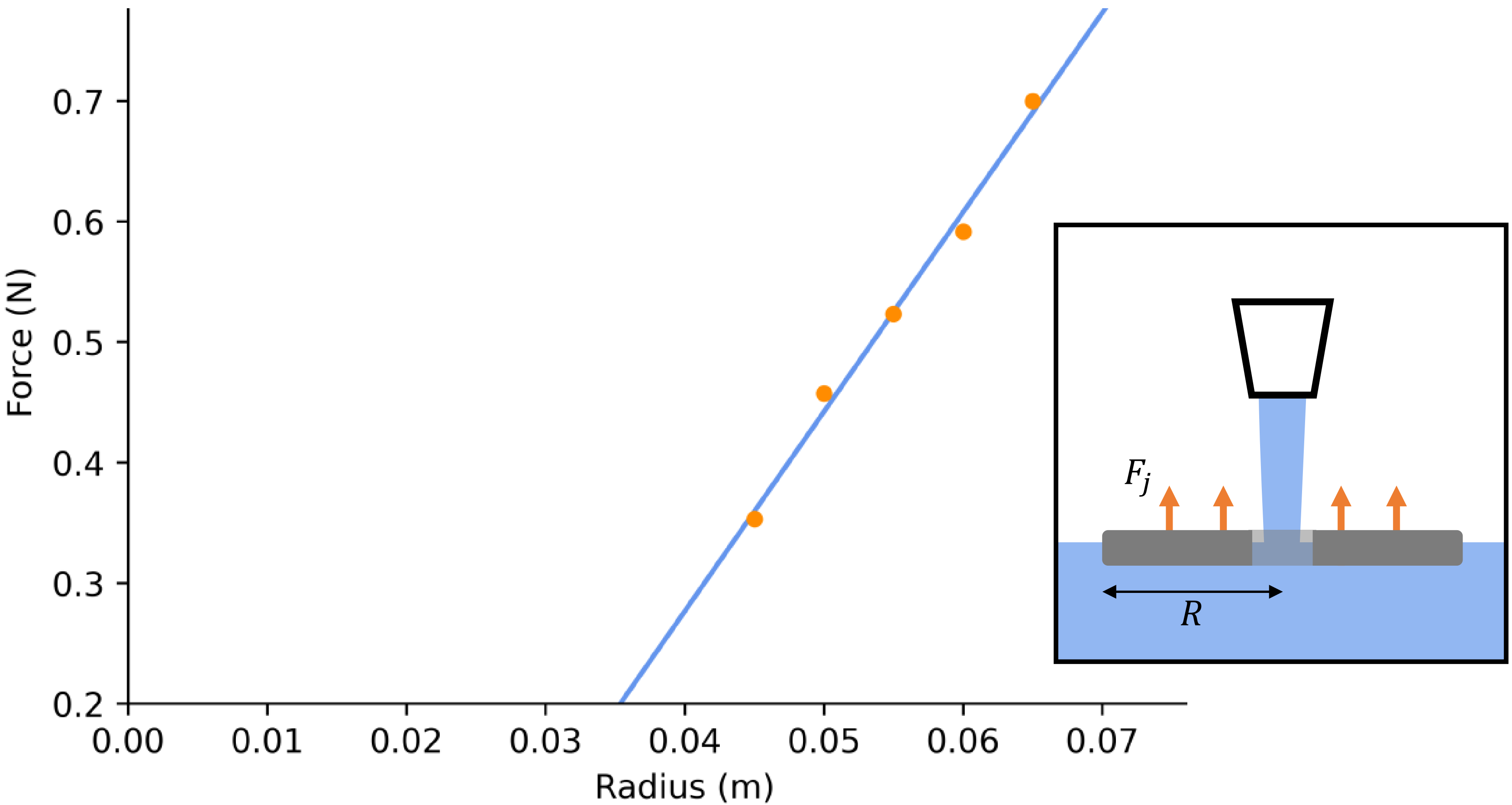


Float **Sink**

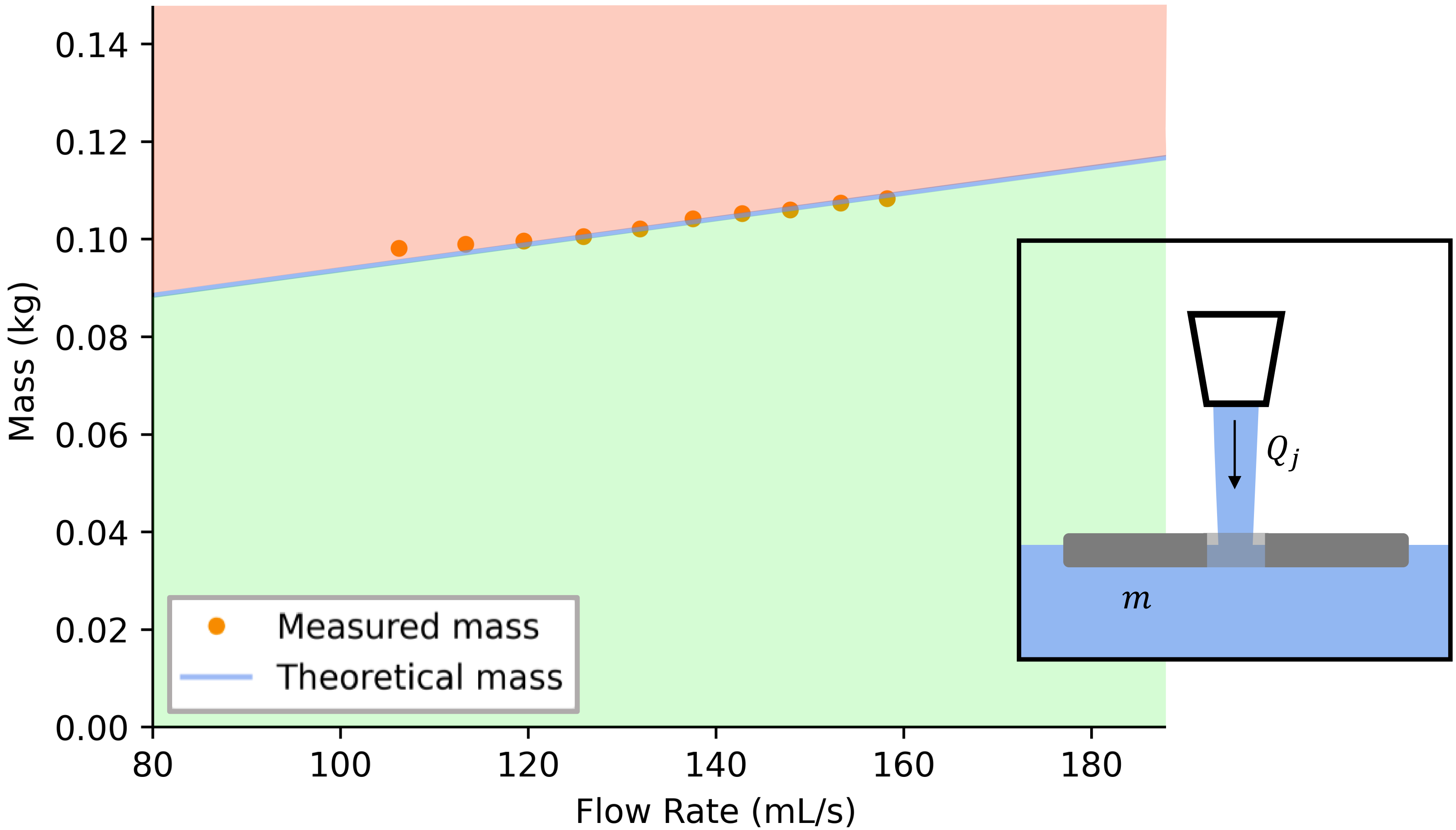
Jet Force vs. Flow Rate



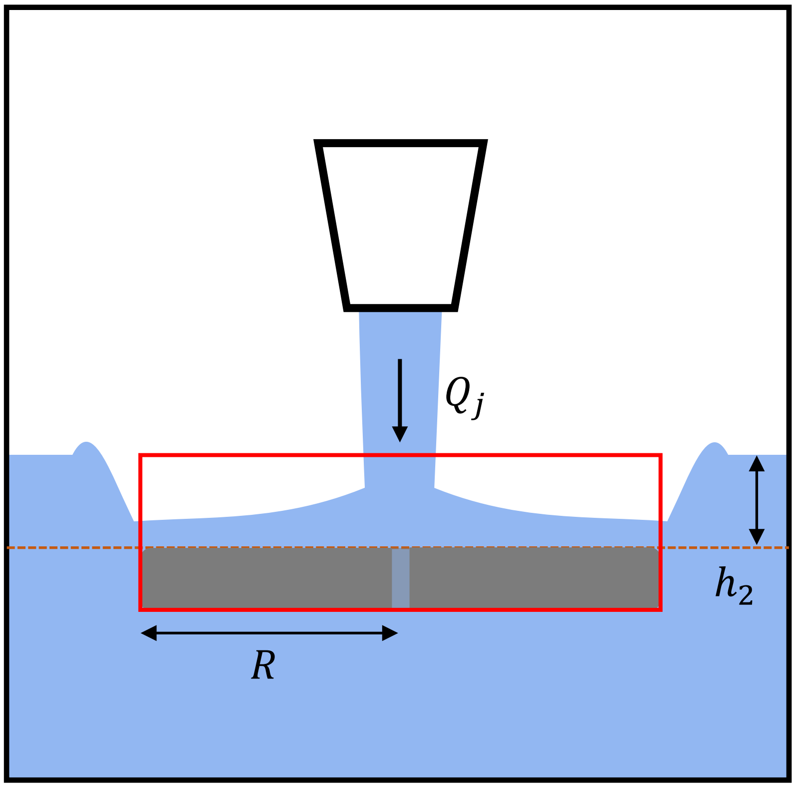
Jet Force vs. Disk Radius



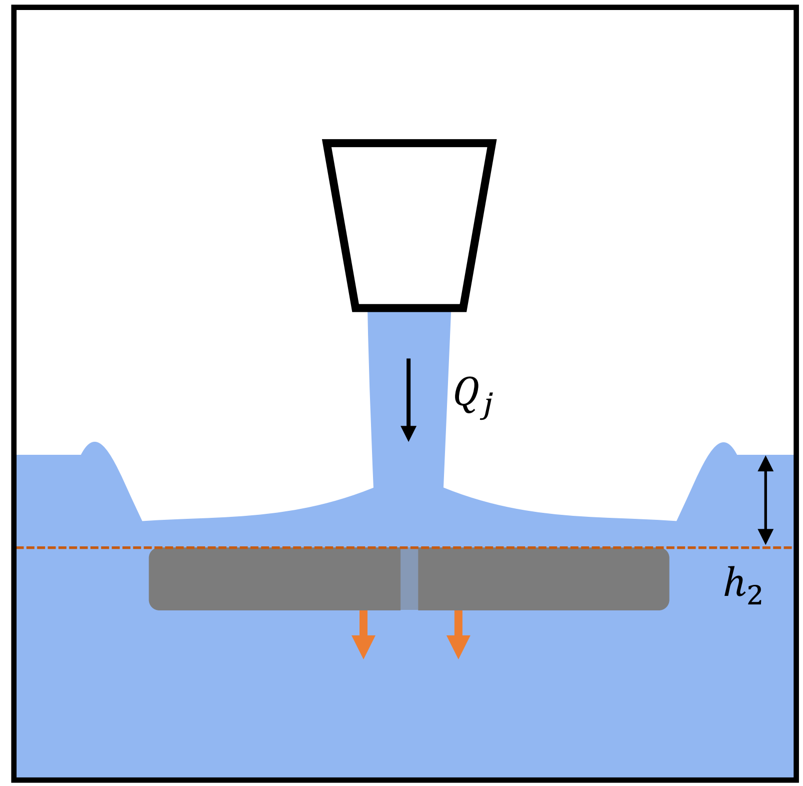
Maximum Mass vs. Flow Rate



Key Parameters – Hydraulic Jump

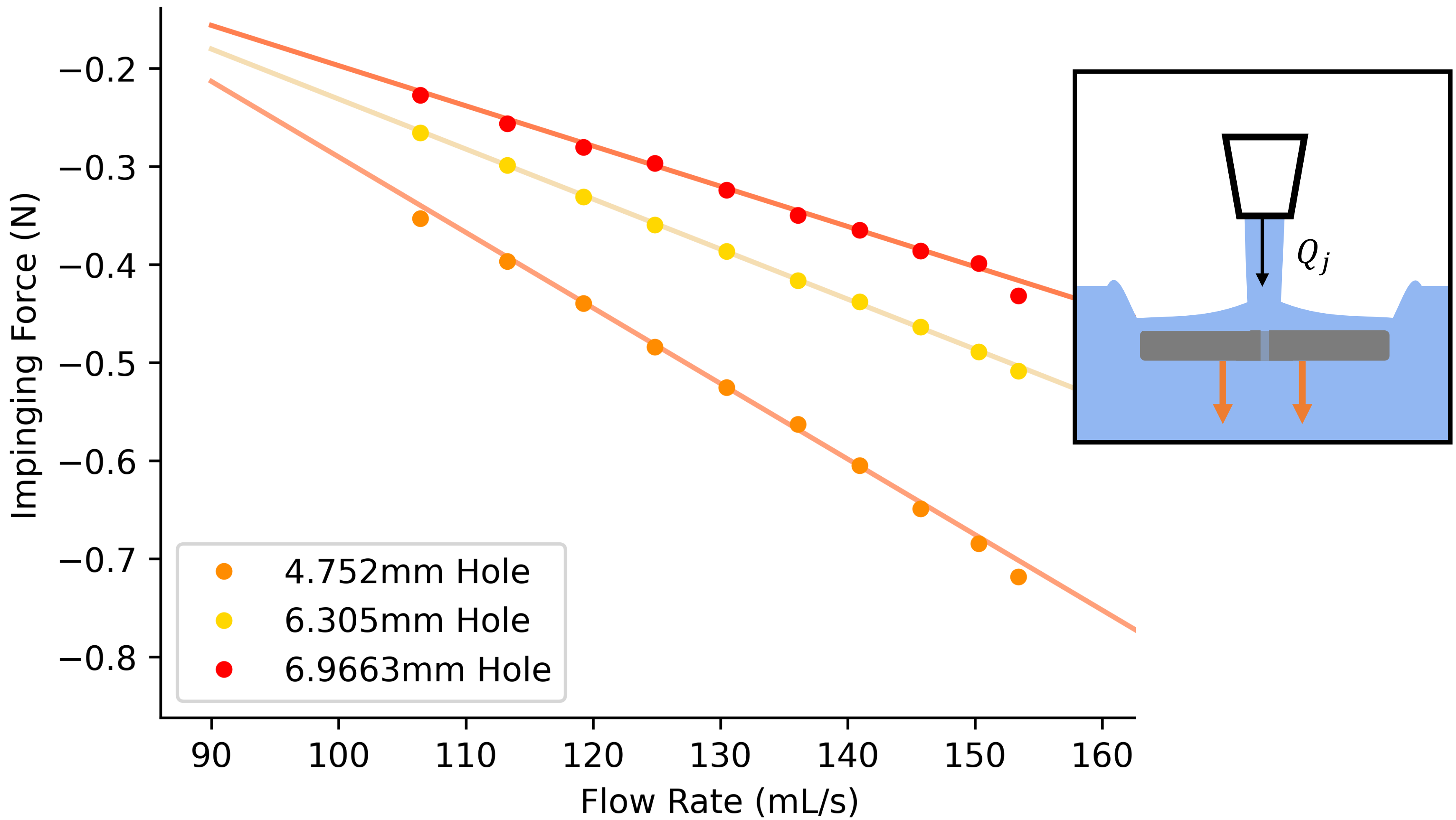


Buoyant Force

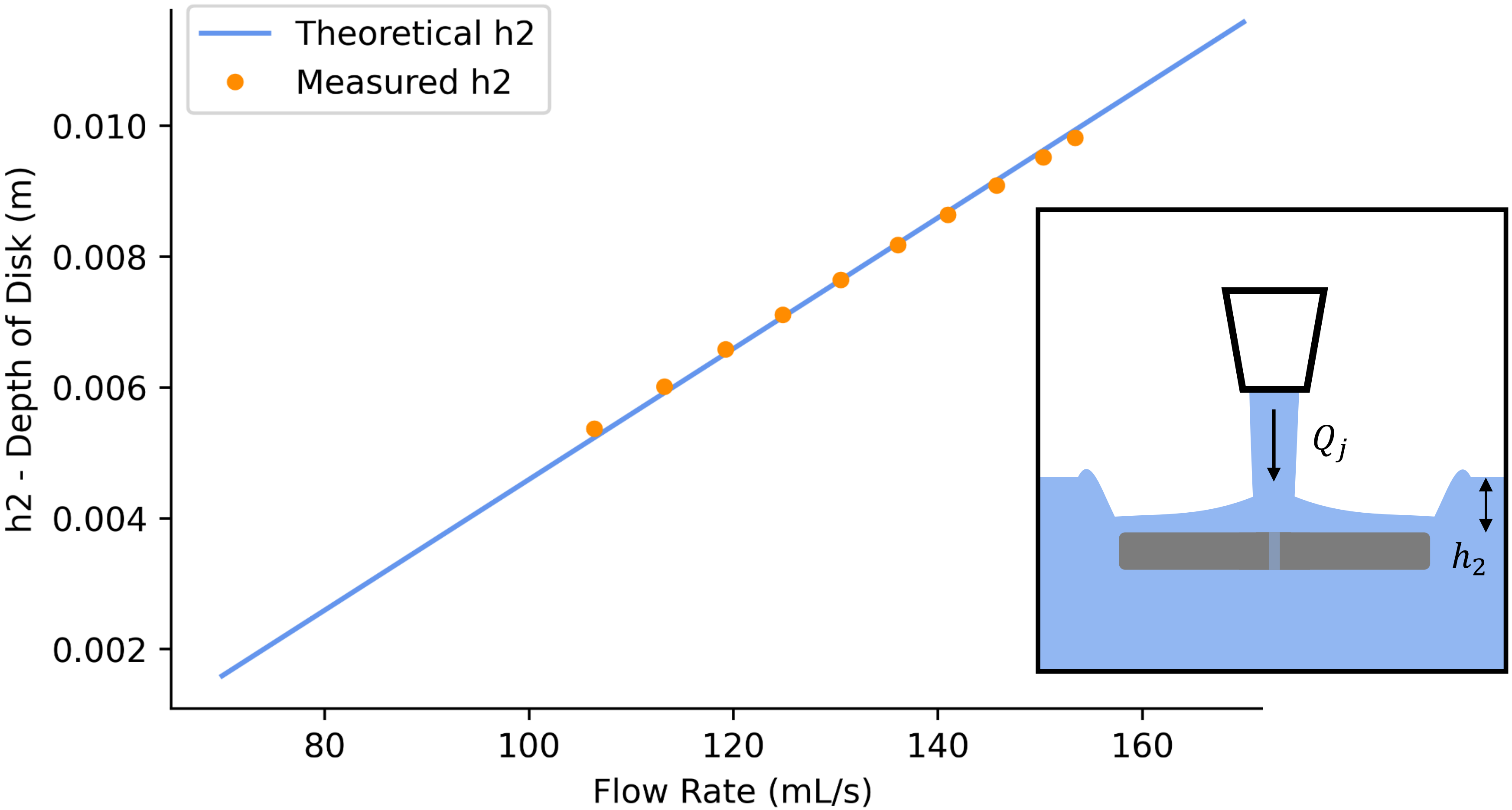


Impinging Force

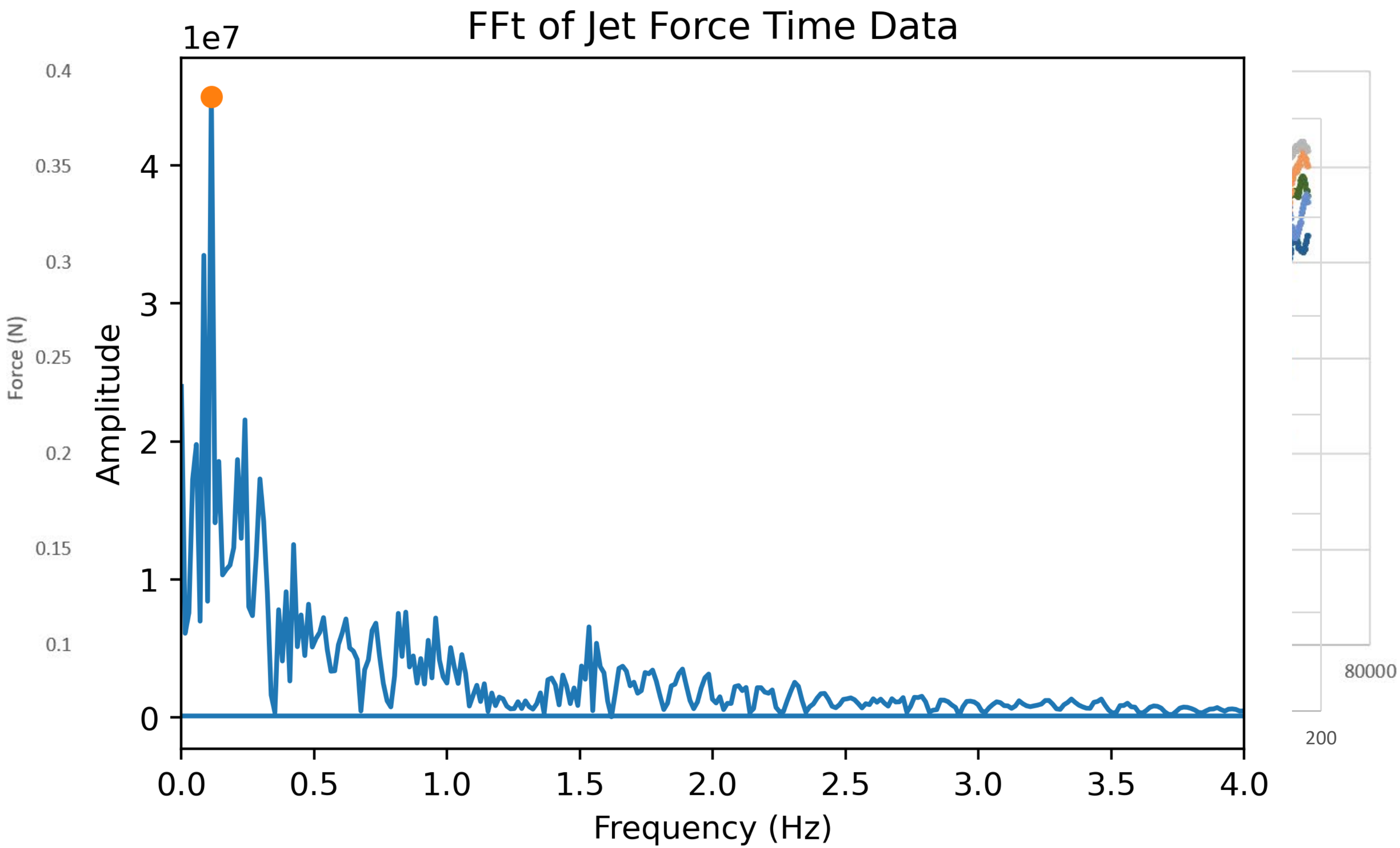
Impinging Force vs Flow and Hole



Disk Depth (buoyancy) vs. Flow Rate



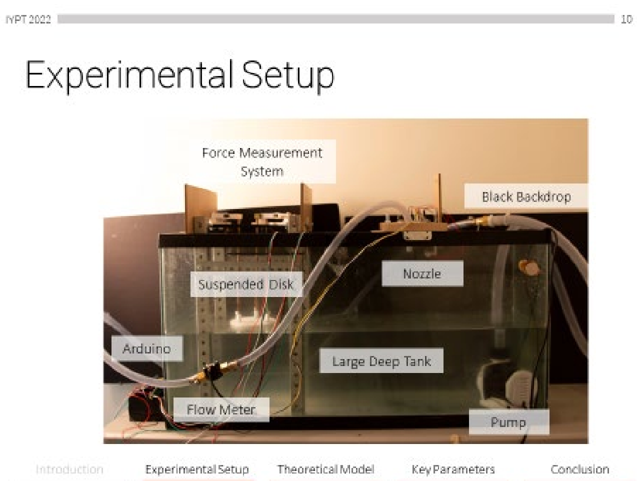
Further Insights

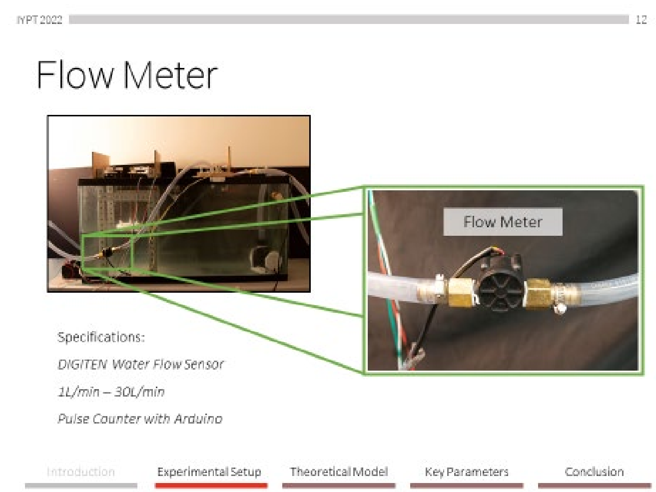


Conclusion

*"A metal disk with a **hole** at its centre **sinks** in a container filled with **water**. When a **vertical water jet** hits the **centre of the disc**, it may **float** on the water surface. **Explain** this phenomenon and investigate the **relevant parameters**."*

Controlled experimental setup

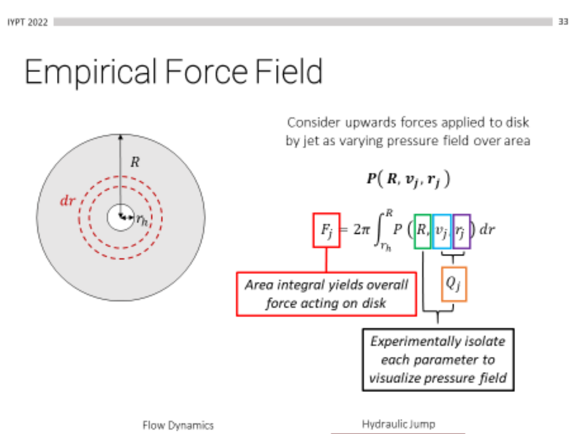


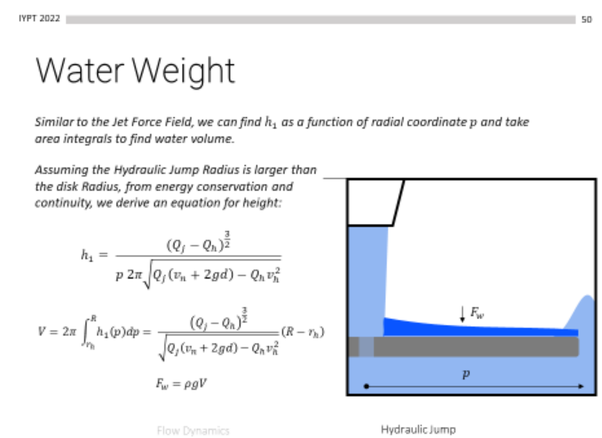


Conclusion

“A metal disk with a ***hole*** *at its centre* ***sinks*** *in a container filled with* ***water****. When a* ***vertical water jet*** *hits the* ***centre of the disc****, it may* ***float*** *on the water surface.* ***Explain*** *this phenomenon and investigate the* ***relevant parameters****.”*

Thorough theoretical model for both cases

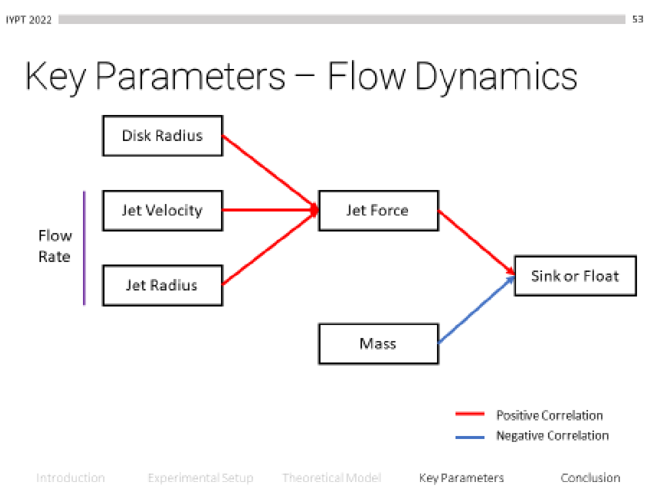


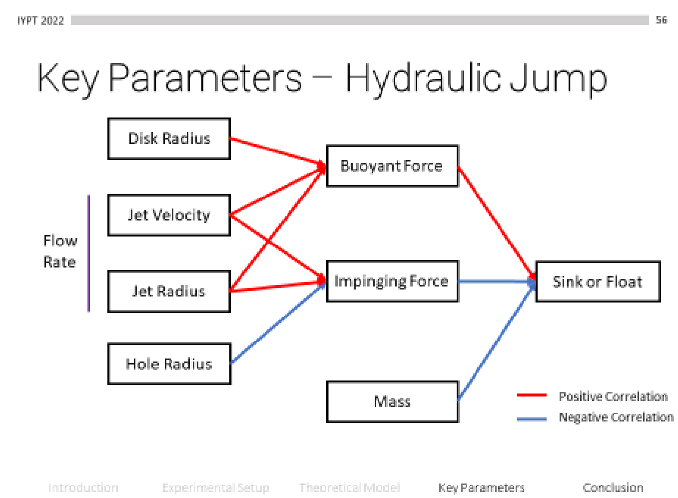


Conclusion

"A metal disk with a ***hole*** *at its centre* ***sinks*** *in a container filled with* ***water****. When a* ***vertical water jet*** *hits the* ***centre of the disc****, it may* ***float*** *on the water surface.* ***Explain*** *this phenomenon and investigate the* ***relevant parameters****."*

Varied key parameters with experimental verification





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Thank you for listening

Appendix

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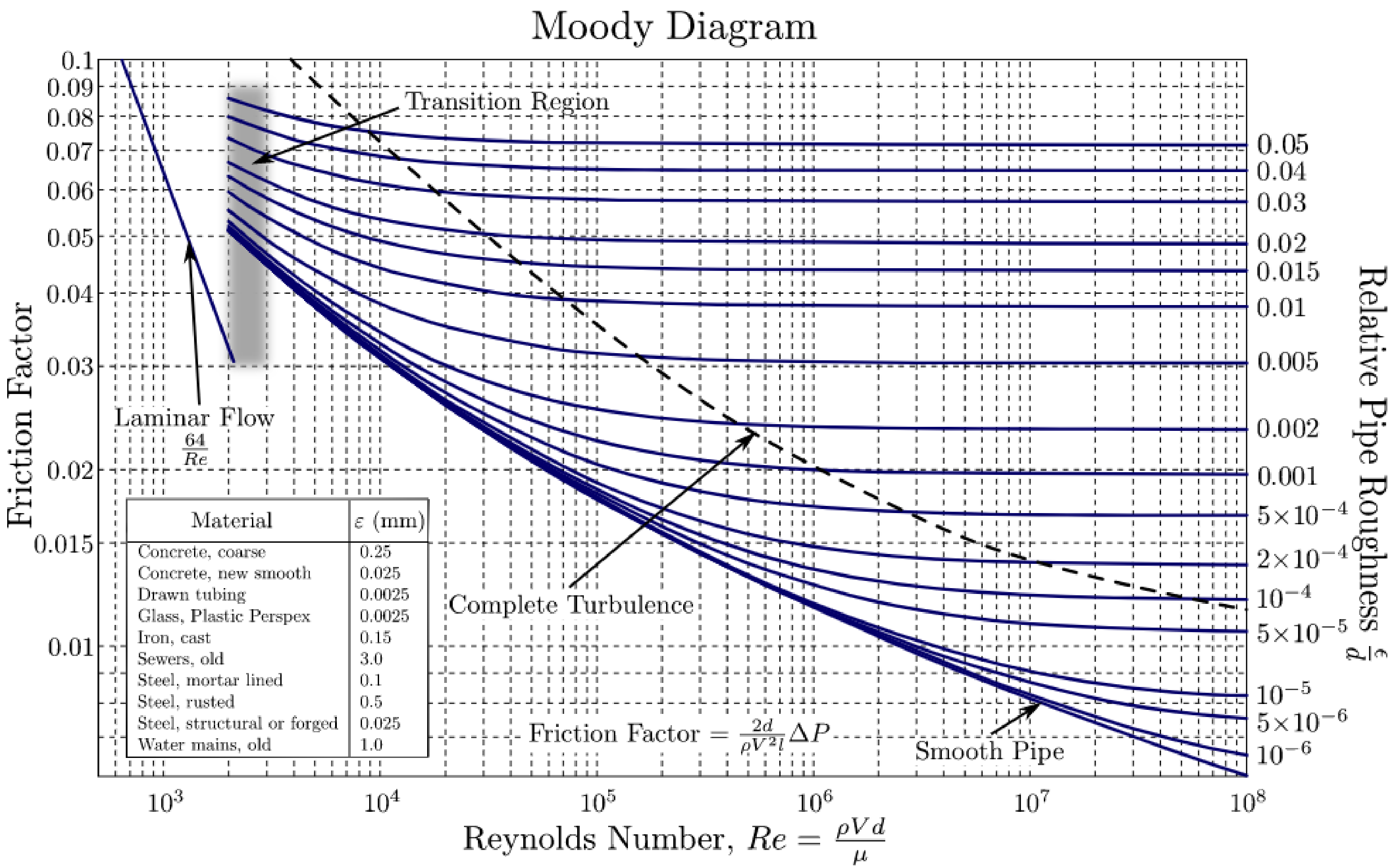
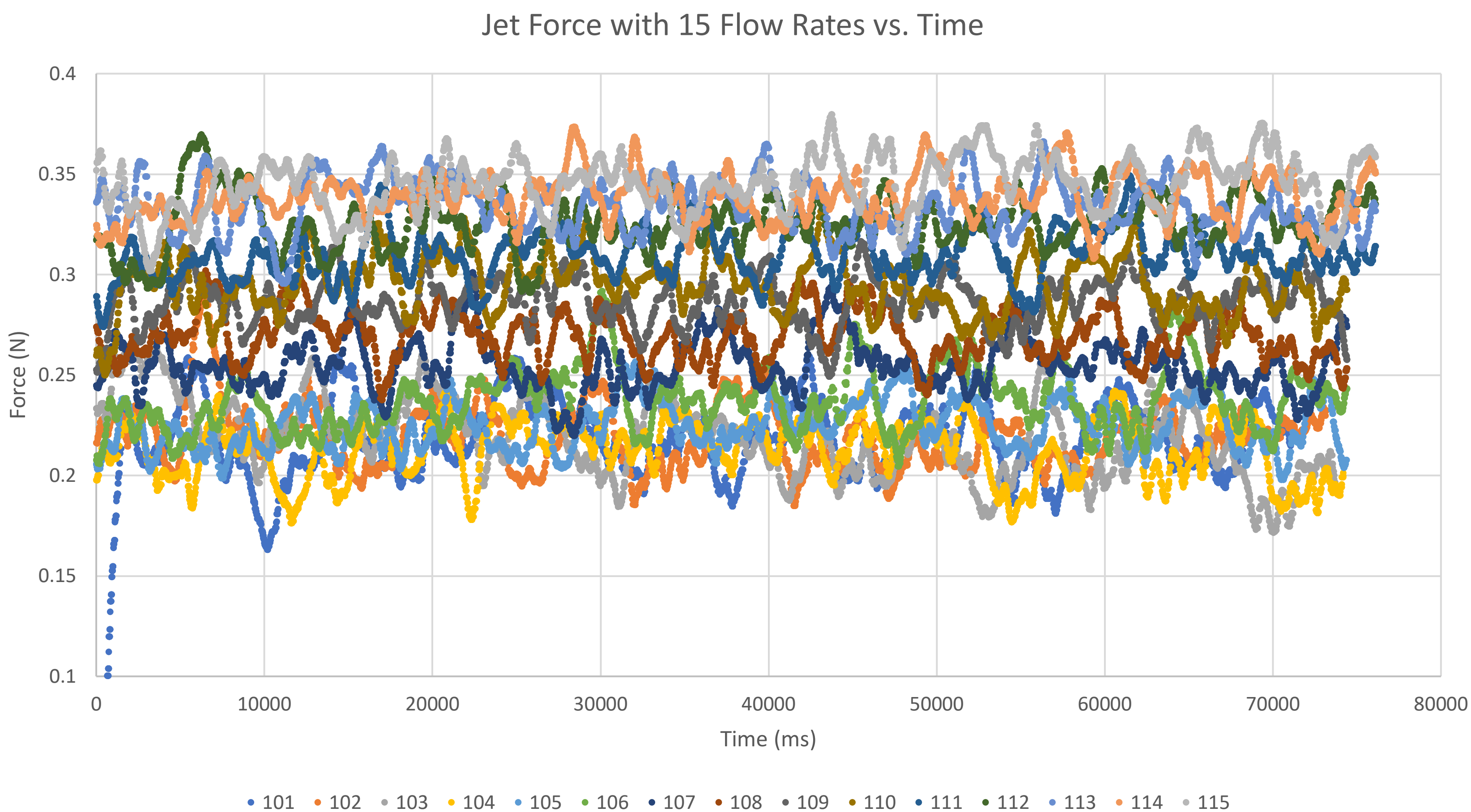
Appendix A: Moody Diagram



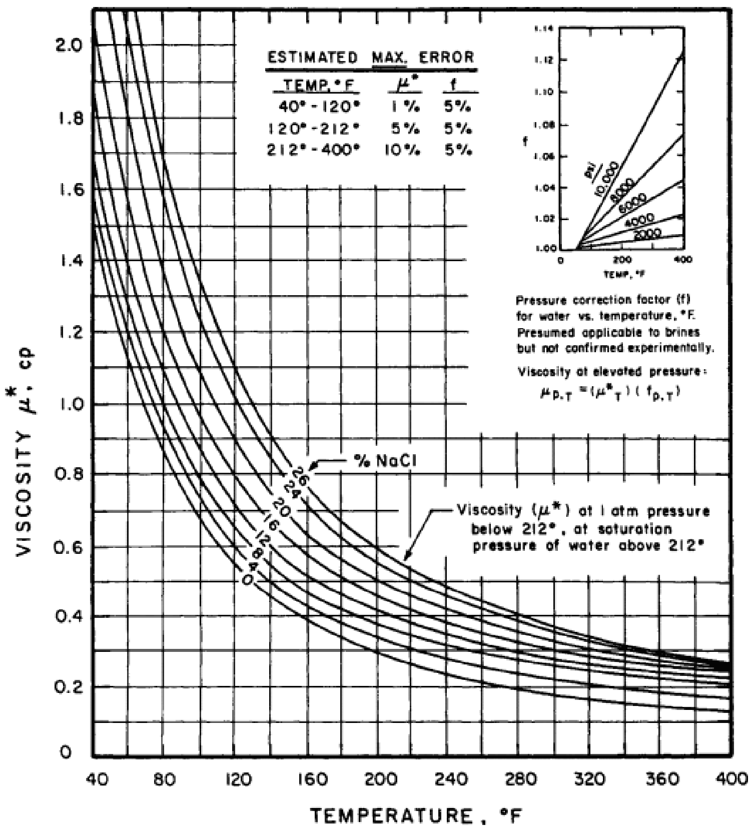
Appendix B: Consistent Force Measurement



Appendix C: Flow Meter Code

```
void loop() {
  interrupts();   //Enables interrupts on the Arduino
  if (restart == true){
    ini_time = micros();
    restart = false;
  }
  if (count >= 5){
    time_elapsed = micros() - ini_time;
    flowRate = 1000000 * count * slope / time_elapsed;
    Serial.print("flow rate:  ");
    Serial.print(flowRate);
count = 0;
    restart = true;
  }
}
```

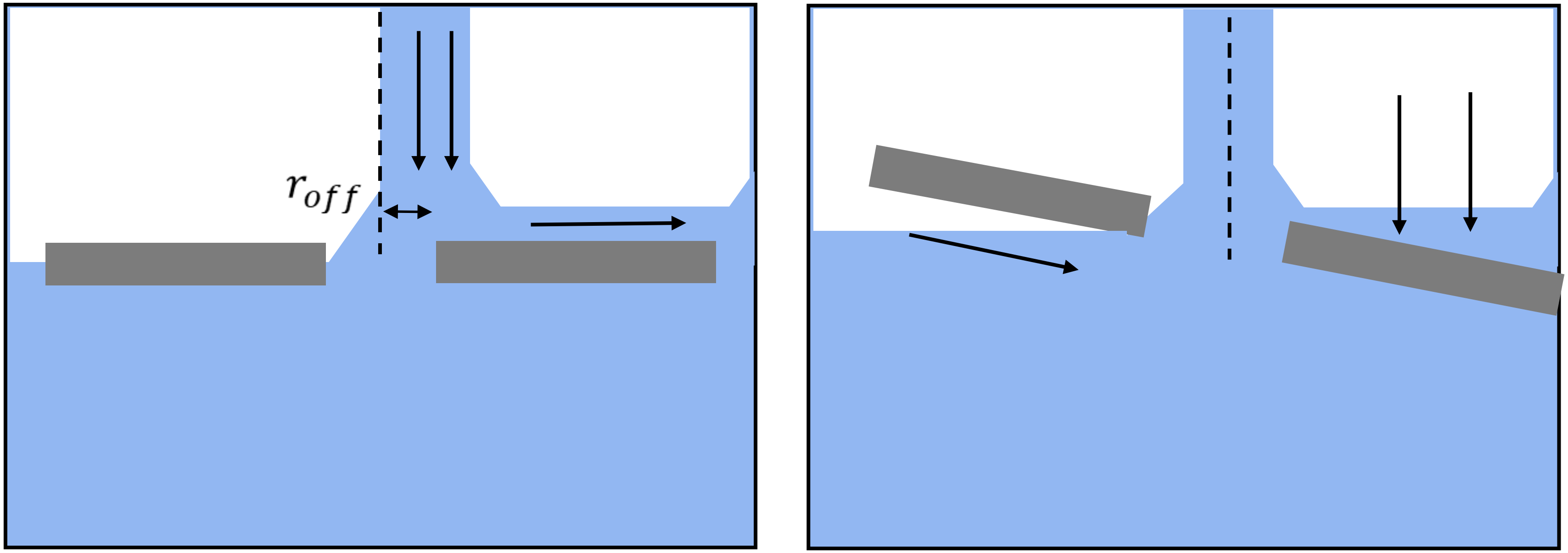
Appendix D: Temperature and Density of Fluid



Solution %(Mass-Volume)	Temperature	Density	Viscosity
0	20	0.9982	1
8	27	1.0559	1
16	35	1.1162	1
20	43	1.1478	1
26	55	1.193	1

Appendix F: Stability Analysis

! *Jet does not collide with edges of disk*



Uneven force distribution from un-centered flow causes torque on one side of disk.
Causes higher edge of disk to shift towards stream, correcting back towards steady state

Appendix G: Torque Calculation

From previous calculation of force colliding on disk:

$$F_a = v_1^2 \rho \pi r_{jet}^2 - v_2^2 \rho A_2$$

$$\Gamma = r_{off} \times F_a = r_{off} F_a sin90°$$

If $\Gamma > \Gamma_b + \Gamma_s$ *, disk will tip over and sink*

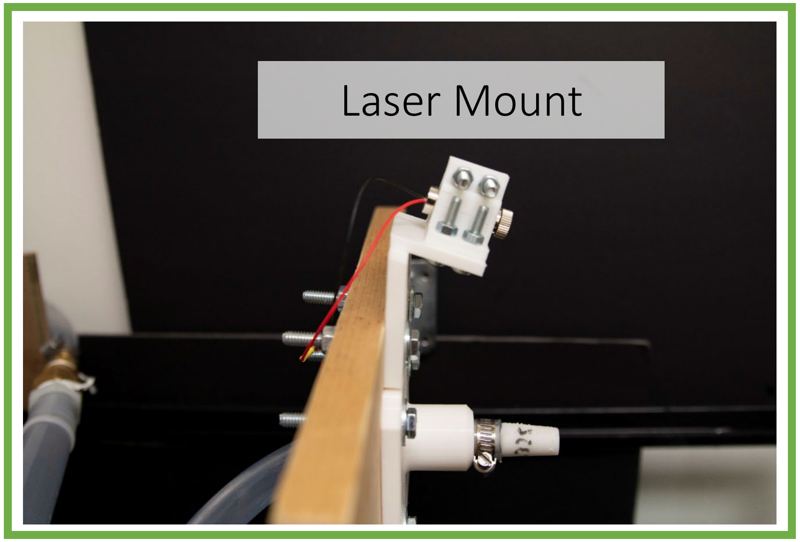
Critical condition:

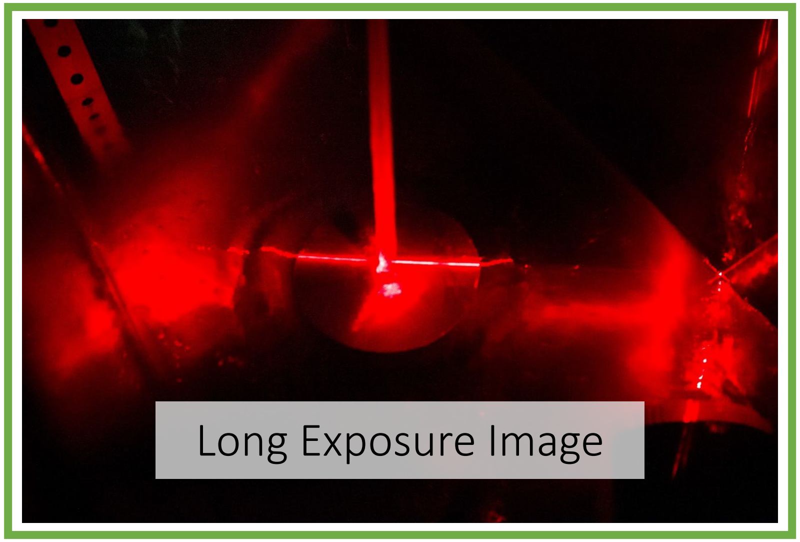
$$r_{off} F_a sin90° = RF_b sin90° + RF_s sin90°$$

r_{off} *scales linearly with disk radius*

$$\boxed{r_{off} = \frac{RF_b + RF_s}{F_a}}$$

Appendix H: Laser Mount





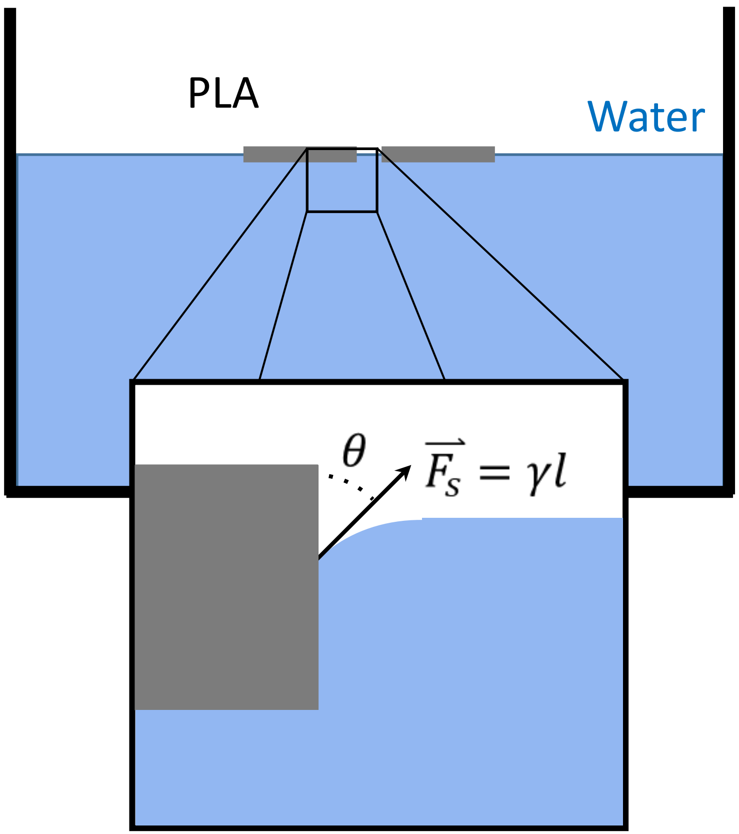
Specifications:

PLA printed 70 degree mount

Long Exposure Images remove turbulence from image

To measure radius and visualize hydraulic jump

Appendix I: Surface Tension, Density, Pressure and Viscosity



From literature, @ 20.9°C:

$$\gamma = 0.0435\,\frac{\mathrm{N}}{\mathrm{m}}$$
$$\theta = 67.7°$$

(Biresaw & Carriere, 2001)

$$\rho = 998.2\,\frac{\mathrm{kg}}{\mathrm{m}^3}$$

(SImetric, 2015)

$$P_{atm} = 101.325\ \mathrm{kPa}$$

(Engineering Toolbox, 2004)

$$\mu = 0.9775 \cdot 10^{-3}\ \mathrm{Pa} \cdot \mathrm{s}$$

(IAPWS, 2008)

Appendix K: Bernoulli Assumptions

- ✓ Inviscid Fluid
- ✓ Steady fluid flow
- ✓ Fluid is incompressible

Appendix L: Calculating Reynolds Number

$$Re = \frac{\rho v L}{\mu}$$

$\rho = density$

$v = flow\ speed$

$L = characteristic\ linear\ dimension$

$\mu = dynamic\ viscosity\ of\ the\ fluid$

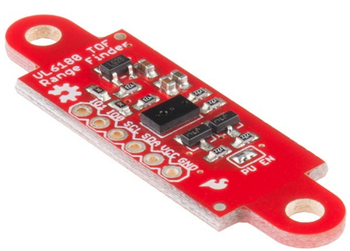
$Re\colon\ 25000 - 35000$

Appendix M: Measuring Physical

Flow Meter ± 0.7%

Analytical Balance ± 0.01g

5kg Load Cell + HX711 Amplifier



VL6180 Range Sensor: ± 0.5mm

Digital Caliper ± 0.02mm

Canon EOS 100D – DSLR Camera